1708

File Cy:

BRL CR 323

# BRL

AD A035171

CONTRACT REPORT NO. 323

# THE RESPONSE OF THIN TARGETS TO PROJECTILE IMPACT

Prepared by

Systems, Science and Software
P. O. Box 1620
La Jolla, California 92037

December 1976

Approved for public release; distribution unlimited.

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | |
|---|---|
| 1. REPORT NUMBER<br>CONTRACT REPORT NO. 323 | 2. GOVT ACCESSION NO. |
| 3. RECIPIENT'S CATALOG NUMBER | |
| 4. TITLE (and Subtitle)<br>THE RESPONSE OF THIN TARGETS TO PROJECTILE IMPACT | 5. TYPE OF REPORT & PERIOD COVERED<br>Final Report |
| | 6. PERFORMING ORG. REPORT NUMBER<br>SSS-R-75-2630 |
| 7. AUTHOR(s)<br>J. K. Dienes | 8. CONTRACT OR GRANT NUMBER(s)<br>DAAD05-75-C-0724 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Systems, Science and Software<br>P. O. Box 1620<br>La Jolla, California 92037 | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>USA Ballistic Research Laboratory<br>Aberdeen Proving Ground, MD 21005 | 12. REPORT DATE<br>DECEMBER 1976 |
| | 13. NUMBER OF PAGES<br>53 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)<br>US Army Materiel Development & Readiness Command<br>5001 Eisenhower Avenue<br>Alexandria, VA 22333 | 15. SECURITY CLASS. (of this report)<br>UNCLASSIFIED |
| | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

| | | |
|---|---|---|
| Helmet | Personnel Armor | Woven Roving |
| Helmet Penetration | Membrane Theory | XP Plastic |
| Helmet Perforation | Nylon | KEVLAR |
| Residual Velocity | Hadfield Steel | Titanium |
| Ballistic Limit | Glass Fabric | |

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)


The response of thin targets to projectile impact is investigated under the assumption that membrane theory is sufficient to describe the interaction theoretically derived. Residual velocities of penetrating projectiles for a number of materials are compared with experimental data. The method allows data for a specific target thickness, projectile size and projectile velocity to be used in determining the ballistic properties under other impact conditions. This requires that a value for the ballistic figure of merit, W, be obtained for each material, which can then be used to rank a number of


DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. materials in a quantitative fashion, even though the tests performed were quite dissimilar.

TABLE OF CONTENTS

# I. INTRODUCTION AND SUMMARY

The response of thin targets to projectile impact is investigated under the assumption that membrane theory is sufficient to describe the interaction. Though it is possible with existing computer codes to compute such problems in greater detail as described, for example, by Dienes et al[1], the membrane theory is amenable to analytic treatment since the equation of motion is of only second order. This makes it possible to gain an overview of the problem which exhibits the main dynamic phenomena. It also makes it possible to analyze approximately the response of materials such as cloth and skin which are not represented by bending theory either in analytic or in computer solutions.

The organization of this report is strongly influenced by a companion document by Dienes and Miles[2] which describes the membrane theory in detail in a form suitable for publication in the open literature. In this report the main conclusions are summarized, and applied to a variety of ballistic problems. In Section II the one dimensional plastic response of a wire to impact is analyzed. The conclusions are similar to those for axisymmetric impact, but the mathematics is considerably simpler because of the elementary nature of solutions to the one dimensional wave equation. The results of the axisymmetric impact problem are summarized in Section III. One of the main conclusions is that the residual velocity does not go to zero as the impact velocity decreases to the ballistic limit. Projectiles that penetrate will always do so with a substantial velocity. Residual velocities of penetrating projectiles are compared with experimental data in Section IV. The method allows data for a specific target thickness, projectile size and projectile velocity to be used in determining the ballistic properties under other impact conditions. This requires that a value for the ballistic figure of merit, w, be obtained for each material. Since the resistance of materials to impact is determined by this parameter, it can be used to rank materials in a quantitative fashion, even when the tests performed are quite dissimilar. It is shown in Section V that this approach works successfully for a nylon for which low speed data is available. At high speeds, however, penetration at the ballistic limit may involve cratering, melting, spallation and other phenomena which the current theory does not allow for. In Section VI, the figure of merit, w, is given for a number of materials. This allows them to be ranked in order of their ballistic figure of merit. Finally, conclusions and recommendations are given in Section VII.

In Appendix I it is shown that membrane theory correlates well with experimental data on the response of circular plates. A listing of the computer program used to evaluate the ballistic limit and residual velocity is given in Appendix II.

# II. ONE DIMENSIONAL IMPACT

To illustrate the physical character of the membrane approach to target deformation, consider the response of a rigid, perfectly plastic wire to impact. Since the tension in the wire is constant, its motion is governed by the equation

$$\rho A\ddot{y} = \sigma A y'' \tag{2.1}$$

where A denotes the cross-sectional area of the wire; $y(x,t)$, its deflection; $\rho$, the density and $\sigma$ the stress. In view of the assumption of ideal plasticity

$$\sigma = \zeta Y \tag{2.2}$$

where Y is the flow stress and $\zeta$ is $\pm$ 1, the stress being positive when the strain rate is positive and negative otherwise. Consequently, the equation of motion reduces to the classical wave equation

$$\ddot{y} = c^2 y'' \tag{2.3}$$

where

$$c = \sqrt{Y/\rho}$$

during the initial phase of the impact, with the quantity, c, being interpreted as the speed of plastic waves.

The solution can be written in the form

$$y = f(x - ct) + g(x + ct). \tag{2.4}$$

We consider only the right going waves in the region $x > 0$ of Fig. 1, so that the solution reduces to the form

$$y = f(x - ct). \tag{2.5}$$

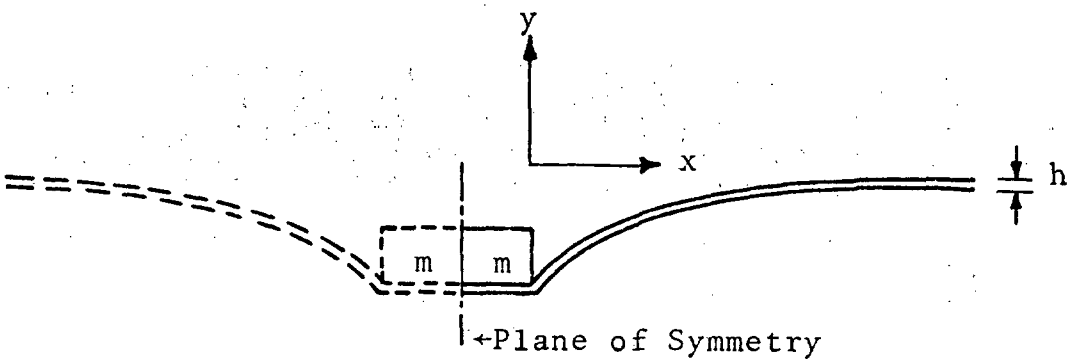


Fig. 1 Sketch of geometry for the problem of an infinite, ductile wire struck by a mass, m.

Only the region to the right of the projectile need be considered, since the geometry is symmetric with respect to the center of the impacting mass. The boundary condition to be imposed at the origin is

$$m\ddot{z} = hYy' \tag{2.6}$$

where m denotes the mass per unit width and

$$z = y(o,t). \tag{2.7}$$

denotes the displacement of the mass. In view of Eq. 1.5 the boundary condition can be written

$$\alpha y' = y'' \tag{2.8}$$

where

$$\alpha = h\sigma/c^2 m = h\rho/m. \tag{2.9}$$

The argument of y in Eq. 1.8 is

$$\xi = - ct \tag{2.10}$$

since the equation is valid only at x = 0. It has the solution

$$f(\xi) = y_0 (e^{\alpha \xi} - 1) \tag{2.11}$$

under the constraint that the deflection must initially vanish. It is straightforward to show that the overall string response is, then, given by

$$y = y_0 (e^{\alpha (x - ct)} - 1) . \tag{2.12}$$

The initial condition for this problem, based on conservation of momentum, is

$$(m + \rho ha)\dot{y} = mv \tag{2.13}$$

where v is the initial projectile velocity. Solving for the unknown coefficient, one finds

$$y = \frac{m}{m + \rho ha} \frac{m}{h\rho} \frac{v}{c} \left(1 - e^{\alpha (x - ct)}\right) , \quad x < ct \tag{2.14a}$$

$$= 0. \qquad , \quad x > ct \tag{2.14b}$$

At late times the deflection approaches the value

$$y_\infty = \frac{m}{m + h\rho a} \frac{m}{h\rho c} v. \tag{2.15}$$

Using the small strain approximation

$$\varepsilon = \frac{1}{2} y'^2 \tag{2.16}$$

we find the maximum strain equal to

$$\varepsilon_{max} = \frac{1}{2}\left(\frac{m}{m + \rho h a}\,\frac{v}{c}\right)^2 . \tag{2.17}$$

If it is assumed that the material is ductile and fails when the strain reaches a critical value, $\varepsilon_f$, (sometimes termed the breaking index) then failure occurs when the impact velocity exceeds the critical value

$$v_c = \frac{m + \rho h a}{m}\sqrt{2\varepsilon_f Y/\rho} . \tag{2.18}$$

The quantity

$$w = \sqrt{2\varepsilon_f Y/\rho} \tag{2.19}$$

is characteristic of the material and can be determined either from static measurements or from a measurement of the critical velocity, $v_c$. For a rigid-plastic material $w^2/2$ is equal to the amount of energy per unit mass that the material can absorb without failure.

## III. AXISYMMETRIC IMPACT

The response of an infinite target impacted by a cylindrical projectile, under the assumption that the flow stress is constant in the target material, is discussed in detail in the companion report by Dienes and Miles.[2] In this section the main results are summarized with an emphasis on their physical interpretation.

The equation of motion for the deflection, y, of a membrane under the assumption of axial symmetry is

$$\rho y_{tt} = Y(ry_r)_r \tag{3.1}$$

where Y is the flow stress; ρ, the density and r, the radial coordinate. On the contact circle the membrane stresses cause a deceleration of the projectile which leads to the boundary condition

$$2\pi aYhy_r = (m + \pi a^2\rho h)y_{tt} \tag{3.2}$$

where m denotes the projectile mass; a, its radius and h is the membrane thickness. It proves convenient to define the dimensionless variables

$$\eta = r/a \tag{3.3}$$

$$\tau = ct/a \tag{3.4}$$

and

$$\zeta = y/(1 - \mu)\beta a \tag{3.5}$$

where

$$\mu = \frac{\pi a^2\rho h}{m + \pi a^2\rho h} \tag{3.6}$$

and

$$\beta = v/c \,. \tag{3.7}$$

In terms of these dimensionless variables the boundary value problem can be cast into the form

$$(\eta\zeta_\eta)_\eta = \eta\zeta_{\tau\tau} \tag{3.8}$$

with the boundary condition at $\eta = 1$

$$2\mu\zeta_\eta = \zeta_{\tau\tau} \tag{3.9}$$

and the initial conditions

$$\zeta = \dot{\zeta} = 0 \qquad (\eta > 1,\ \tau = 0) \tag{3.10}$$

and

$$\zeta' = 0,\ \dot{\zeta} = 1 \qquad (\tau = 0) \,. \tag{3.11}$$

By the method of Laplace transforms it can be shown that the solution can be expressed as

$$\zeta = 4\mu \ \mathrm{Re}\left[Ae^{s_0\tau}\right] + 2\mu J(\tau) \tag{3.12}$$

where Re denotes the real part of the expression in brackets. Here

$$A = \frac{1}{s_0^3 + 4\mu(1 - \mu)s_0} \tag{3.13}$$

and

$$J(\tau) = \int_0^\infty \frac{e^{-x\tau}dx}{[x^2K_0(x) + 2\mu K_1(x)]^2 + \pi^2[x^2I_0(x) - 2\mu I_1(x)]^2} \tag{3.14}$$

where $K_0$, $K_1$, $I_0$ and $I_1$ are modified Bessel functions. In the expression for A, $s_0$ denotes the root of the transcendental equation

$$sK_0(s) + 2\mu K_1(s) = 0, \tag{3.15}$$

The term, A, arises from the contribution of a pair of poles in the s plane. The maximum dimensionless deflection $\zeta_{max}$, was obtained numerically and is shown in Fig. 2.

The strain in the membrane is given, as in the previous section, by

$$\varepsilon = \frac{1}{2} y_r^2 \tag{3.16}$$

for small deformations. Denoting by $a$ the deceleration of the projectile, it can be readily shown that the strain is given by

$$\varepsilon = \frac{1}{2} (1 - \mu)^2(\beta a)^2 . \tag{3.17}$$

Under the assumption of a critical strain failure criterion, it follows that penetration occurs for impact velocities above

$$v_c = \frac{w}{(1 - \mu) a_m} \tag{3.18}$$

where $a_m$ denotes the maximum value of $a$. As in the previous section, the quantity w can be taken as a figure of merit and its value is given by

$$w = \sqrt{2\varepsilon_f Y/\rho} \tag{3.19}$$

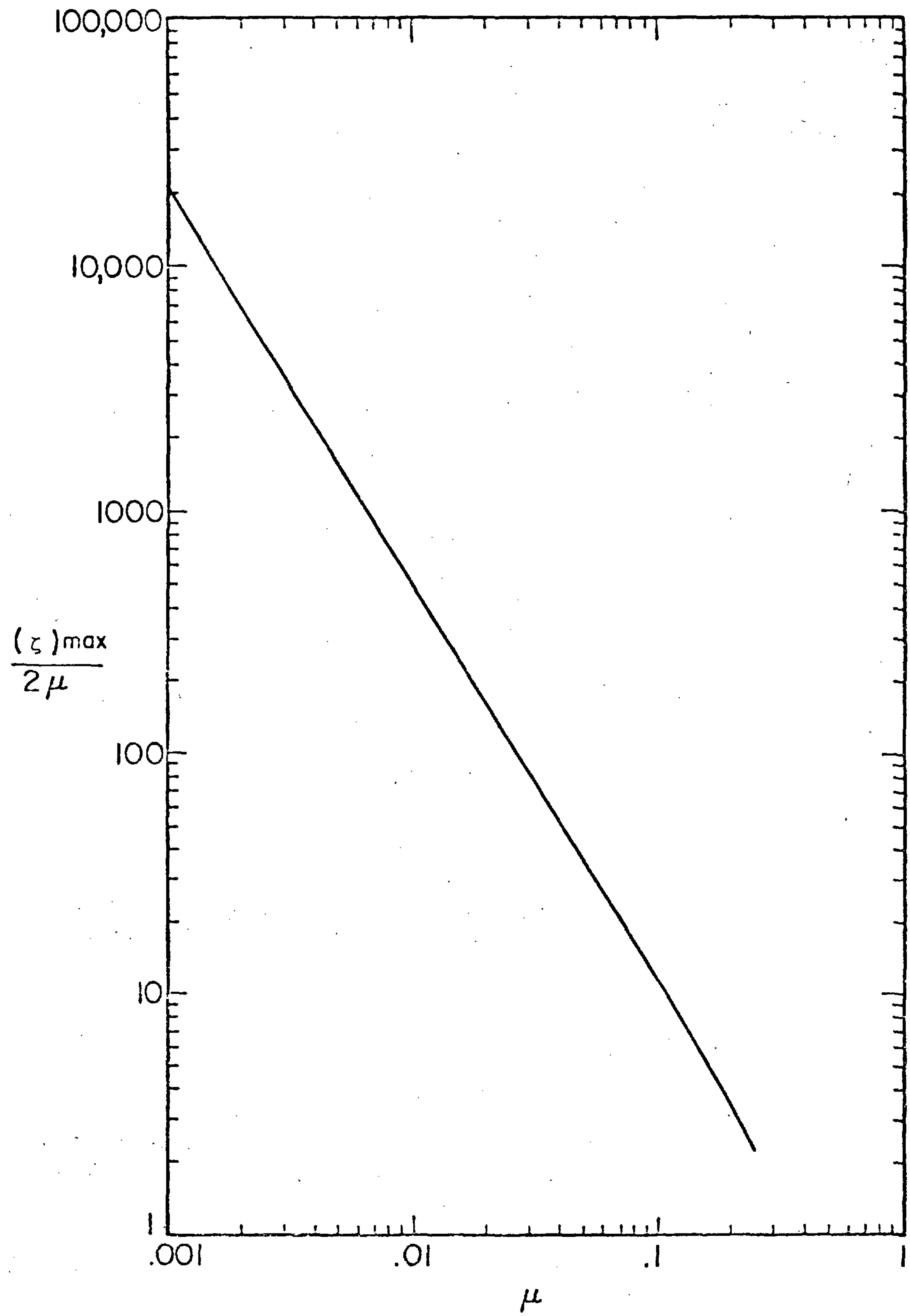


Fig. 2. The maximum value of the dimensionless deflection. The actual peak deflection is given by $y_{max} = (1-\mu)\beta a\zeta_{max}$.

The quantity $a_m$ is plotted in Fig. 3 as a function of $\mu$. Its value always exceeds unity. Thus, the ballistic limit for a cylinder of radius a is always less than that for a rectangular prism of width a. The ballistic limit decreases as $\mu$ becomes small which, from inspection of (3.6), occurs when the projectile length increases and the radius decreases with a fixed projectile mass. This is, of course, in agreement with one's expectations. The dimensionless time at which peak deflection occurs, $\tau_d$, is shown in Fig. 4, which also includes the dimensionless time to maximum strain, $\tau_m$. The actual times, based on Eq. 3.4, are

$$t_d = a\tau_d/c \tag{3.21}$$

for maximum deflection, and

$$t_m = a\tau_m/c \tag{3.22}$$

for maximum strain. Maximum strain always occurs prior to maximum deflection. This implies that when failure occurs, based on a maximum strain criterion, that the projectile is still in motion and, hence, that it will penetrate at a finite velocity. It is shown in Ref. 2 that the residual velocity is

$$v_r = F(\mu)v,$$

with $F(\mu)$ being plotted in Fig. 5.

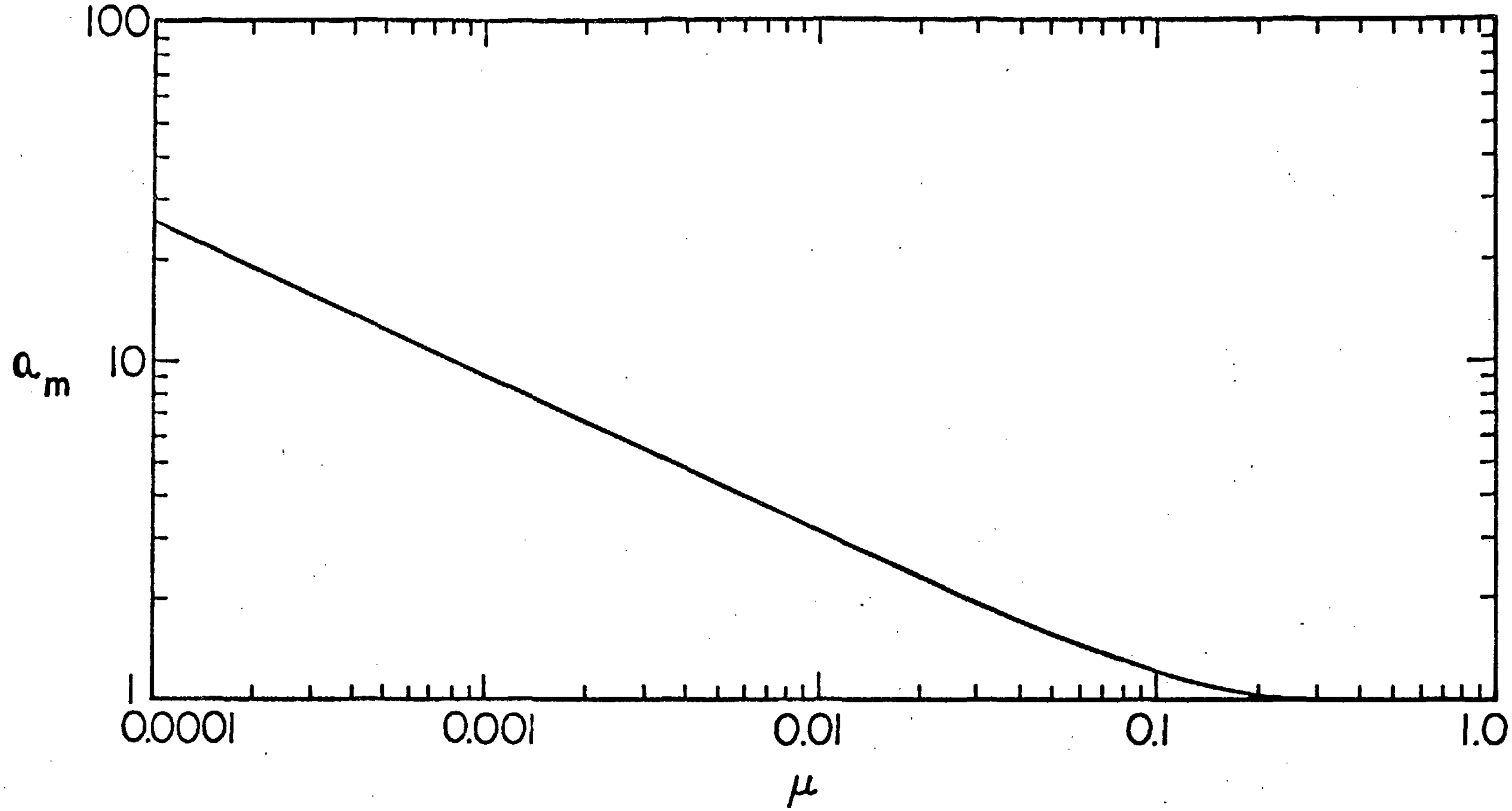


Fig. 3. The maximum value of the normalized deceleration, $a_m(\mu) = a(\tau_m)$; $\tau_m$ is plotted in Fig. 4.

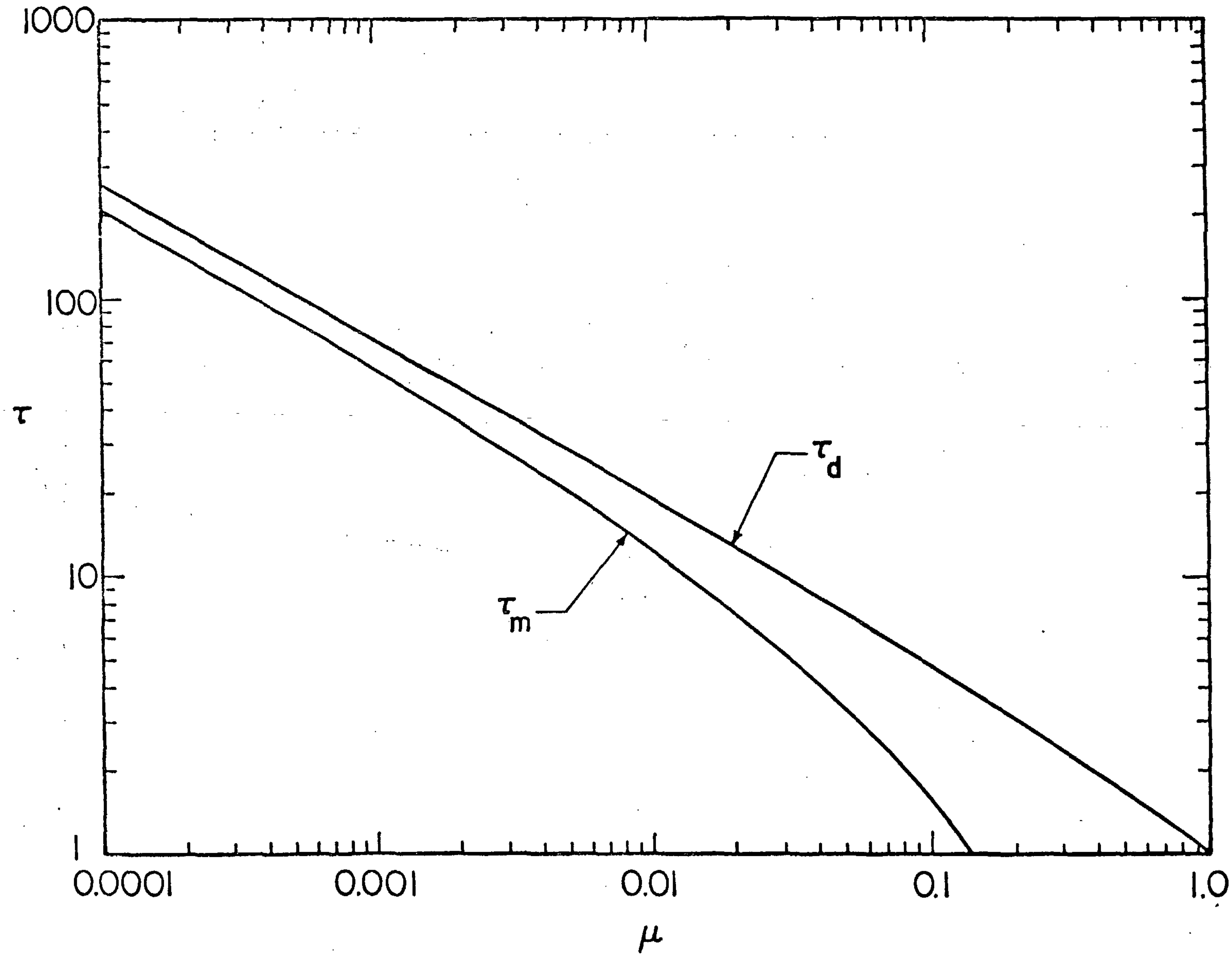


Fig. 4. The time $\tau_d$ at which the deflection is a maximum and the time $\tau_m$ at which the strain is a maximum. Note that maximum strain always precedes maximum deflection.

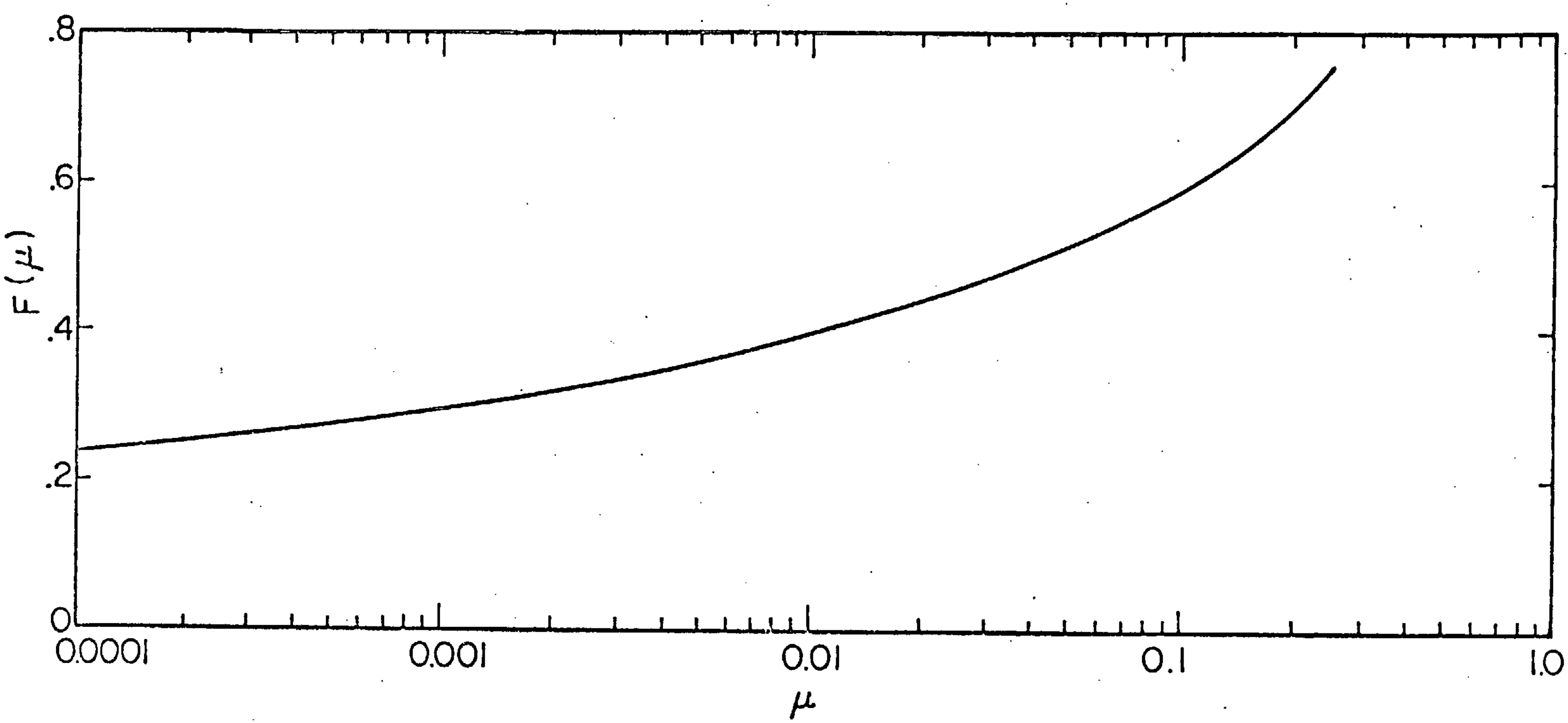


Fig. 5. The ratio of the residual velocity to the initial velocity of the projectile.

## IV. COMPARISON OF MEMBRANE THEORY WITH EXPERIMENT

It is known that for small deflections, particularly when the materials lie in the elastic regime, the deformation of plates is governed by bending theory. With increasing deflections, stretching of the middle surface becomes important and the membrane stresses begin to dominate. Ultimately, for very large deflections, the resistance is governed virtually in its entirety by the membrane stresses, which may be considered uniform through the thickness and independent of radius. A comparison between calculations based on this approach and measured deflections of impulsively loaded circular metal plates is given in Appendix I. The comparison shows that membrane theory describes the deformation well when the deflection exceeds 15% of the radius.

To indicate the potential accuracy of the method for determining the ballistic limit, we consider here the resistance of a sheet of the titanium alloy Ti5Al-2.5Sn. The residual velocities measured in a series of tests described by Bruchey[3] in which steel cylinders were fired into sheets 0.1295 cm in thickness are shown in Fig. 6 for a cylinder of radius .284 cm and mass 1.037 gm. Based on a titanium density of 4.5 g/cm$^3$ we find $\mu = 0.125$ and $F(\mu) = 1.11$. If we take the ballistic limit from Fig. 6 as the lowest striking velocity at which residual velocities are clustered, about 340 m/s, then $w = 330$ m/s. Now, the flow stress for a similar alloy, Ti6Al-4V, is given by Lindholm, Yeakley and Bessey [4] as 188ksi ($13.0 \times 10^9 d/cm^2$) and the cited strain to failure based on post-test measurements is 18 percent. (The similarity in properties of the alloys can be verified by comparing stress-strain curves given by Wolf[5]). Using these data we find $w = 324$ m/s, which is only 2 percent less than the value obtained from ballistic data. The strain-to-failure of 18% based on post-test measurements is signigicantly larger than is indicated by their stress-strain curve, but such data is not commonly available in handbooks, and for this reason we have had to refer to the more specialized data for Ti6Al-4V.

The minimum residual velocity can be obtained using the value of $\mu$ cited above and Fig. 7. For a projectile velocity of 340 m/s we find $v_r = 210$ m/s, in good agreement with the lowest values observed by Bruchey. This result, that the residual velocity does not go to zero as the impact velocity decreases to the ballistic limit, is our most striking conclusion.

To determine the residual velocity above the ballistic limit, we note that at $r = a$ the strain is given by Eq. 3.16.

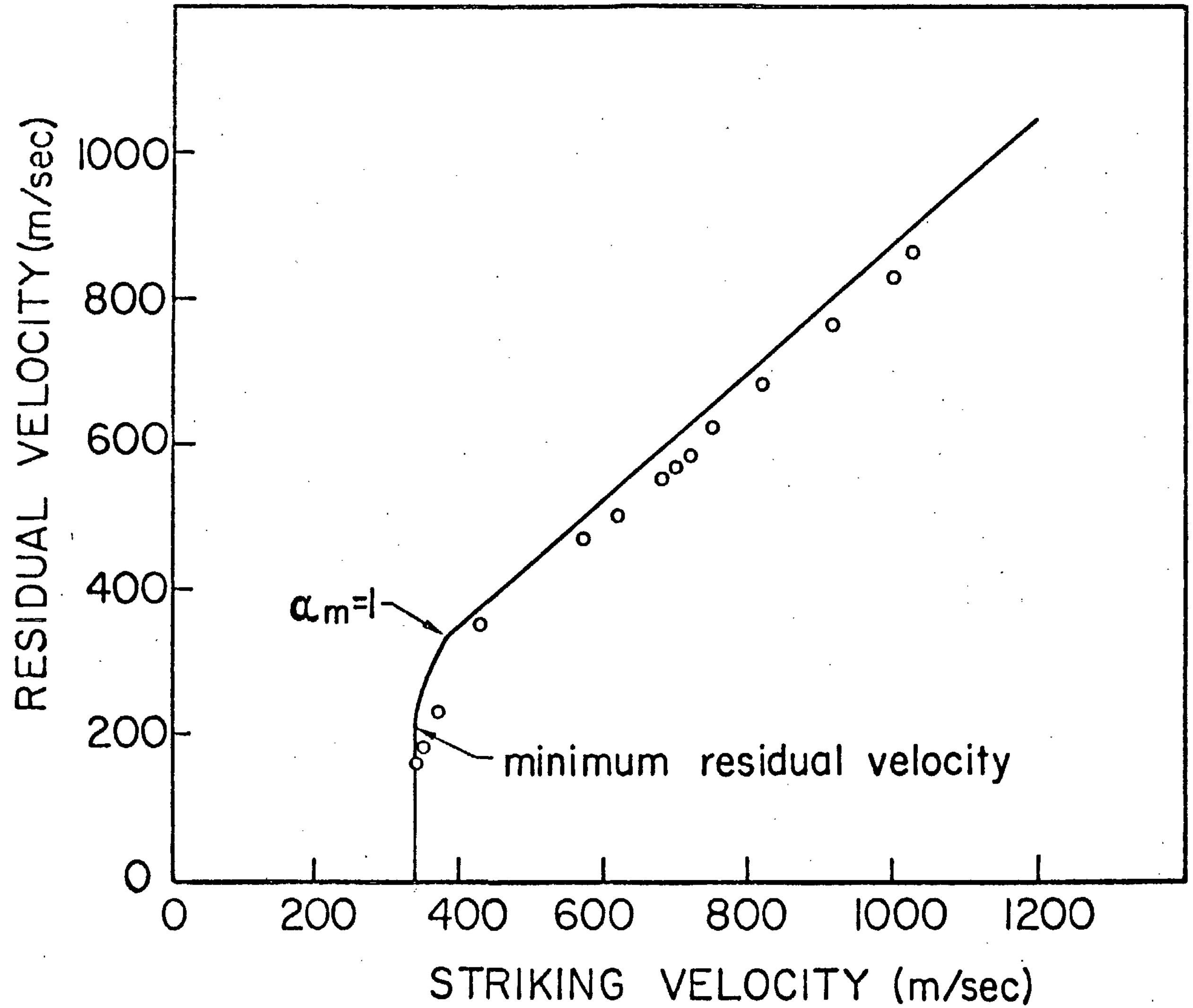


Fig. 6. A comparison of theoretical and measured residual velocities for a 1.04gm steel cylinder fired into an 0.13 cm (thick) titanium plate (Bruchey 1973). Failure occurs without membrane deflection for velocities above that at which $a_m(\mu) = 1$.

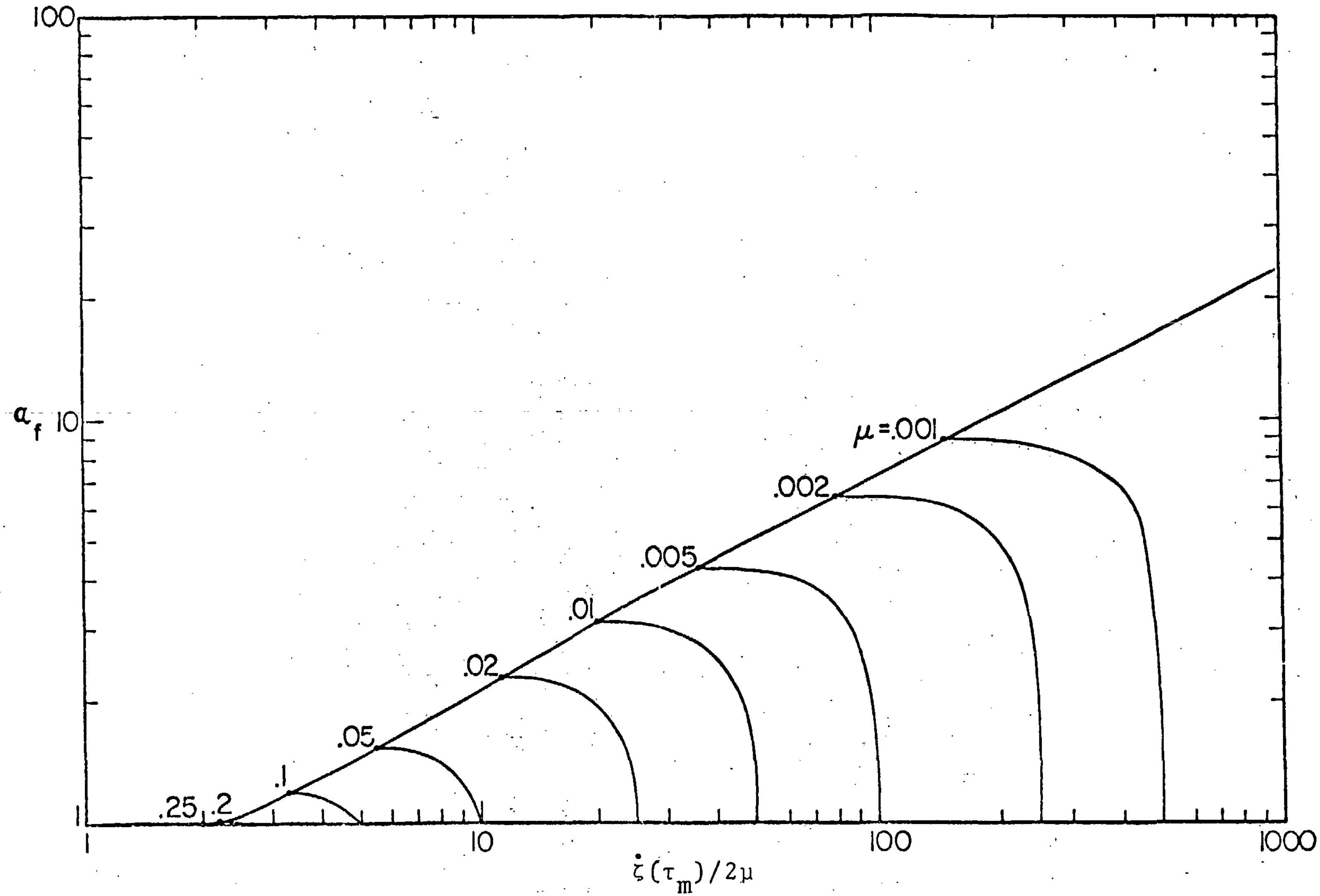


Fig. 7. The relation between $\dot{\zeta}_m = \dot{\zeta}(\tau_m)$ and $a_m \equiv a(\tau_m)$ for various values of the parameter $\mu$; see (3.17), (4.2) and (4.3). The upper boundary is the locus of $a_f = a_m$ and determines the residual velocity at the ballistic limit. For $a_f = 1$ plug failure occurs without membrane deformation.

The value of $y_r$ at the time of maximum strain may be obtained by solving

$$\sqrt{2\varepsilon_f} = -(1-\mu)\beta\ddot{\zeta}(\tau_m) \tag{4.1}$$

for the time of maximum strain, $\tau_m$, at a specific impact velocity (recalling that $\ddot{\zeta} < 0$). Then the residual velocity is given by

$$v_r = 2\mu(1-\mu)v\dot{\zeta}(\tau_m) \ . \tag{4.2}$$

In Fig. 7 we have plotted $a_f = -\ddot{\zeta}(\tau_m)/2$ versus $\zeta(2\mu)/2\mu$ for various values of $\mu$. Thus, $\dot{\zeta}(\tau_m)$ can be obtained from (4.1) without explicitly determining $\tau_m$ by means of the graphs. In practise, it is convenient to determine $a_f$ from the relation

$$a_f = \frac{w/v}{2\mu(1-\mu)} \tag{4.3}$$

and the residual velocity is given by 4.2. The residual velocities for $a_f \geq 1$ shown in Fig. 6 were obtained in this way, and though the experimental data are too scattered to confirm the theory in detail in this range, the overall trend is reasonable. As the impact velocity increases, the time to maximum strain decreases until at $\tau_m = 0$, $\ddot{\zeta} = 1$. The corresponding value of v, which we denote by $\overline{v}$, is the upper bound of the impact velocities that result in membrane deflection followed by membrane failure.

At impact velocities above $\overline{v}$ the membrane fails instantly (in the current theoretical model) and the residual velocity is reduced only by transfer of momentum to the ruptured plug that is removed from the membrane, Then

$$v_r = \frac{m}{m + \pi a^2 \rho h} v \tag{4.3a}$$

$$= (1-\mu)v \tag{4.3b}$$

This trend agrees well with the measured residual velocities shown in Fig. 6.

## V. CORRELATION OF DATA FOR DIFFERENT VELOCITIES

The theory outlined in the preceding sections is based on a number of strong assumptions concerning projectile shape and rigidity and the characteristics of the target. In this section we consider two experimental determinations of the ballistic limit and examine the correlation provided by membrane theory.

Laible, Figucia and Ferguson[6] have examined the resistance to penetration of a number of high-modulus fibers, primarily nylons and other organic fibers. In one pair of experiments the ballistic limit ($v_{50}$) was determined for an organic fiber described as X-500, type I. Its tenacity is reported as 12-14 grams per denier; the modulus as 500 grams per denier; elongation to failure as 2-4%; and density as 1.47 g/cm$^3$. The relevance of these properties to the properties of the fabric is somewhat obscure, however, since the mechanical behavior of a fabric may be very complex even when its constituent fibers behave in a linear fashion. The tests involved impact by a 17 grain, 0.22 caliber missile. Its velocity was varied until the value at which 50% of the projectiles were defeated by the target was determined, and this value was defined as the ballistic limit, $v_{50}$. The results are summarized in Table I. It is remarkable that the values of the ballistic figure of merit, w, agree within 2 percent between the two tests in view of the many respects in which the details of the encounter differ from the assumptions made in the analysis.

Bruchey[3] has determined the ballistic limit for two steel projectiles fired into titanium targets of the same thickness. The results for the heavier projectile, with a mass of 1.037 g, were described in the previous section. The projectile radius was 0.284 cm, and the areal density of the target was .583 g/cm$^2$. As previously described, this leads to a value of 330 m/sec for the ballistic figure of merit, w. A cylinder having dimensions which were approximately half those of the low speed projectile (a = .131 cm) was also tested, and the ballistic limit was determined to be 585 m/s. It follows that the value of $\mu$ is .1945, $F(\mu) = 1.02$ and $w = 481$m/s. This figure is significantly higher (46%) than that determined from the low velocity projectile, indicating that at higher speeds the target material behaves more efficiently. This can probably be attributed to the greater dissipation of energy per unit mass involved with high speed projectiles, with a substantial fraction of the energy going into the cratering process. It would be of interest to examine the interaction in careful experiments or numerical calculations to determine in greater detail why the target is more efficient in resisting high speed impacts.

TABLE I

COMPARISON OF THE BALLISTIC RESISTANCE OF TWO THICKNESSES OF X-500

| Areal density oz/ft$^2$ | Areal density, g/cm$^2$ | $\frac{m_1}{\pi a^2 \rho h}$ g | $\mu$ | $F(\mu)$ | $v_{50}$ m/s | $\frac{w}{(1-\mu)F(\mu)v_{50}}$ m/s |
|---|---|---|---|---|---|---|
| 18.4 | .560 | .1369 | .1107 | 1.18 | 326 | 342 |
| 11.1 | .337 | .0824 | .0697 | 1.37 | 263 | 335 |

# VI. RANKING OF MATERIALS

The impact resistance of a number of materials for which ballistic data are available is shown in Table II. Based on this data, the Hadfield steel which has been traditionally used for infantry helmets and many kinds of armor plate ranks lowest among the materials considered with w = 260 m/s. Kevlar, recently developed by Dupont, ranks highest among the materials analyzed, with w = 872 m/s. These data are taken from widely differing sources, however, and it would be desirable to conduct a systematic series of tests to provide a more direct verification of this approach to ranking materials. It is not entirely clear from the raw data what properties constitute a good ballistic material. It may be that woven materials are better than plate materials, and it may be that local melting plays a role in some cases. Though the approach proposed here may serve as a rough guide to the relative efficiency of different materials, the choice of materials for specific protective missions should involve careful consideration of the specific penetration mechanisms.

TABLE II

COMPARISON OF THE BALLISTIC RESISTANCE OF VARIOUS MATERIALS RANKED BY THE PARAMETER w

| Material | Reference | Areal Density, $h\rho$ gm/cm$^2$ | Projectile Mass, m grams | Projectile Radius, a cm | Mass Ratio, $\mu$ $\mu$ | Ballistic Limit, $v_{50}$ meters/sec | w meters/sec |
|---|---|---|---|---|---|---|---|
| Hadfield Steel | McManus[7] | .87 | 1.10 | .279 | .162 | 290 | 260 |
| Glass Fabric | Laible[6] | .55 | 1.10 | .279 | .109 | 282 | 296 |
| Titanium | Bruchey[3] | .58 | 1.04 | .284 | .125 | 340 | 330 |
| Woven Roving | Bruchey[3] | .64 | 1.04 | .284 | .135 | 360 | 342 |
| X-500 Type I | Laible[6] | .56 | 1.10 | .279 | .111 | 326 | 342 |
| Nylon Tire Yarn | Laible[6] | .57 | 1.10 | .279 | .112 | 373 | 391 |
| Aluminized Nylon | Flaherty[8] | .014 | .13 | .159 | .0091 | 132 | 429 |
| Nylon 728 | Bruchey[3] | .41 | 1.104 | .284 | .090 | 485 | 543 |
| XP Plastic | Bruchey[3] | .40 | 1.037 | .284 | .089 | 510 | 576 |
| Kevlar | Kennel[9] | .098 | 10.2 | .483 | .0074 | 244 | 872 |

## VII. CONCLUSIONS AND RECOMMENDATIONS

It has proved possible to determine the motion of a membrane impacted by a cylindrical mass in closed form with the assumption that the flow stress in the membrane material is uniform. A comparison of the computed deflection with measurements published by Florence in 1966 has indicated that the assumption that the flow stress is constant and equal to the yield strength of the material, for circular plates, leads to reasonably accurate results when the deflection exceeds 15% of the radius.

Of the various possible failure theories, a critical strain criterion is the simplest to implement in the current theoretical approach. It appears to lead to good correlation when compared with the low speed titanium results obtained by Bruchey, both with regard to the ballistic limit and the residual velocity. It is found that the residual velocity is finite, even when the target is penetrated at the ballistic limit. This is due to the fact that maximum strain occurs before maximum deflection and, consequently that failure occurs before the motion has stopped. In the example of the low speed titanium impact experiments carried out by Bruchey, the theoretical residual velocity was 210 m/s at the ballistic limit of 340 m/s.

Configurations for which the stress and strain are uniform through the thickness, as assumed in membrane theory, are efficient in comparison with structures that undergo bending stresses, for when loads are resisted by bending moments, the middle surface is strained less than adjacent surfaces, and consequently the material is not being used in an optimum fashion. This may serve as an explanation of a rather surprising result obtained by Laible et al.[6] They find that the addition of resin to fabric laminates lowers the ballistic limit. In one example, the addition of 8.5 oz/ft$^2$ of resin to 10.7 oz/ft$^2$ of fabric lowered the ballistic limit from 830 to 623 ft/sec. This suggests that the filled laminate is subject to bending stress which are not present in the unfilled laminate, and that failure occurs when the bending stress exceeds the allowable value. This occurs at a lower velocity than when the laminates are unbonded, and they are all stressed to the same level. This line of thinking suggests that fabrics are more efficient than plates of the same material. In fact, some tests were run at S$^3$ in which woven wire (340 stainless) was compared with plates of the same material. The woven fabric turned out to have a relatively low ballistic limit. This was subsequently attributed to the fact that the wire had an elongation of only 15%, where the plate had an elongation

of 40%. We believe that if a fabric of wire which was not strain-hardened were fabricated, then it would indeed prove more efficient than plate.

Few materials exhibit a stress-strain relation which closely resembles the rigid-plastic behavior assumed in the current theory. It would be desirable to find a good method for approximating real material behavior by a solution to the linear wave equation. That is, how can one select an equivalent strength, $Y_e$, which in some sense causes the solutions to

$$\rho\ddot{y} = Y(y')\nabla^2 y$$

to be closely approximated by

$$\rho\ddot{y} = Y_e\nabla^2 y$$

Procedures for this kind of analysis, termed the method of equivalent linearization, have been developed for ordinary differential equations, as discussed, for example, by Booten(11) and Dienes,(12), but they do not appear to have been developed adequately for partial differential equations.

The problem of oblique impact has not been solved, even in the simple case of a linear membrane. The separation of the target into regions of tension and compression seems to present a major difficulty. Perhaps it would be best to use a numerical approach to some of these problems, retaining the assumptions of membrane theory, in order to gain a better insight into the mechanics of oblique impact. The results might suggest some approximations that would be useful in the oblique problem.

## REFERENCES


1. Dienes, J. K., M. M. Baligh, L. Morino and J. W. Miles, "Investigation of Helmet Dynamics," Systems, Science and Software Report SSS-R-73-1985, Dec., 1973.

2. Dienes, J. K. and J. W. Miles, "A Membrane Model for High Speed Impact," Systems, Science and Software Report SSS-R-2626, May, 1975.

3. Bruchey, W. J., Jr., "Penetration of Lightweight Personnel Armor Materials by Munition Fragments," BRL interim memorandum report no. 97, April, 1973.

4. Lindholm, U. S., L. M. Yeakley and R. L. Bessey, "An Investigation of the Behavior of Materials under High Rates of Deformation," Air Force Materials Laboratory Report AFML-TR-68-194, July, 1968.

5. Wolf, J. (Coordinating Editor), Aerospace Structural Metals Handbook, Belfour Stulen, Traverse City, Michigan, 1970.

6. Laible, R. C., F. Figucia and W. J. Ferguson, "The Application of High-Modulus Fibers to Ballistic Protection," J. Macromol. Sci.-Chem., A7(1), pp. 295-322, (1973).

7. McManus, L. R., "Protective Helmets of NATO and Other Countries," United States Army Natick Laboratories Report 73-29-CE, Jan., 1973.

8. Flaherty, R. E., "A Study of Low-Velocity Impacts into Thin-Sheet Aluminum and Nylon Cloth," National Aeronautics and Space Administration Report TN D-6324, May, 1971.

9. Kennel, R., Private Communication, Aerospace Corporation, Nov., 1973.

10. Florence, A. L., "Circular Plate under a Uniformly Distributed Impulse," Int. J. Solids and Structures, Vol. 2, pp. 37, 1966.

11. Booten, R. C., Jr., "The Analysis of Non-Linear Control Systems with Random Input," Proceedings, Symposium on Nonlinear Circuit Analysis, Polytechnic Institute, Brooklyn, N. Y., Vol. 2, 1953.

12. Dienes, J. K., "Some Applications of the Theory of Continuous Markoff Processes to Random Oscillation Problems," Ph.D. thesis, California Institute of Technology, 1961.

## APPENDIX I. RESPONSE OF CIRCULAR PLATE TO UNIFORM IMPULSE

Florence[10] has determined the response of both aluminum (6061-T6) and steel (CR-1018) circular plates to sheet explosive, which provides an approximately uniform impulse, and compared the maximum deflection with that calculated from plate (bending) theory. His results, which are shown in Fig. 8, suggest that bending theory is inadequate for deflections that exceed 10% of the plate radius, and he remarked that membrane stresses might account for the discrepancies. We proceed to apply the membrane model to his configuration.

The notation remains as in Section III, except that a is now the radius of the circular membrane. The motion is governed by the axisymmetric wave equation (3.1), the boundary condition (3.2) is replaced by

$$y = 0 \quad (r = a,\ t > 0) \ , \tag{A.1}$$

and the initial conditions are

$$y = 0,\ y_t = v_0 \equiv I/\rho h \quad (r < a,\ t = 0) \ , \tag{A.2a,b}$$

where I is the impulse per unit area. [In reality, $v_0$ is only approximately uniform and falls sharply to zero as $r \uparrow a$ in consequence of the restraint (A.1) at $r = a$; however, this deficiency of the model is relatively unimportant for the prediction of y in $r < a$].

The solution of (3.1), (A.1) and (A.2) is given by

$$y = \sum_{i=1}^{\infty} A_i J_0(\lambda_i \eta) \sin(\lambda_i \tau) \ , \tag{A.3}$$

where $\lambda_i$ is the i'th root of the Bessel function $J_0$,

$$J_0(\lambda_i) = 0 \quad (0 < \lambda_1 < \lambda_2 < \ldots\infty) \ , \tag{A.4}$$

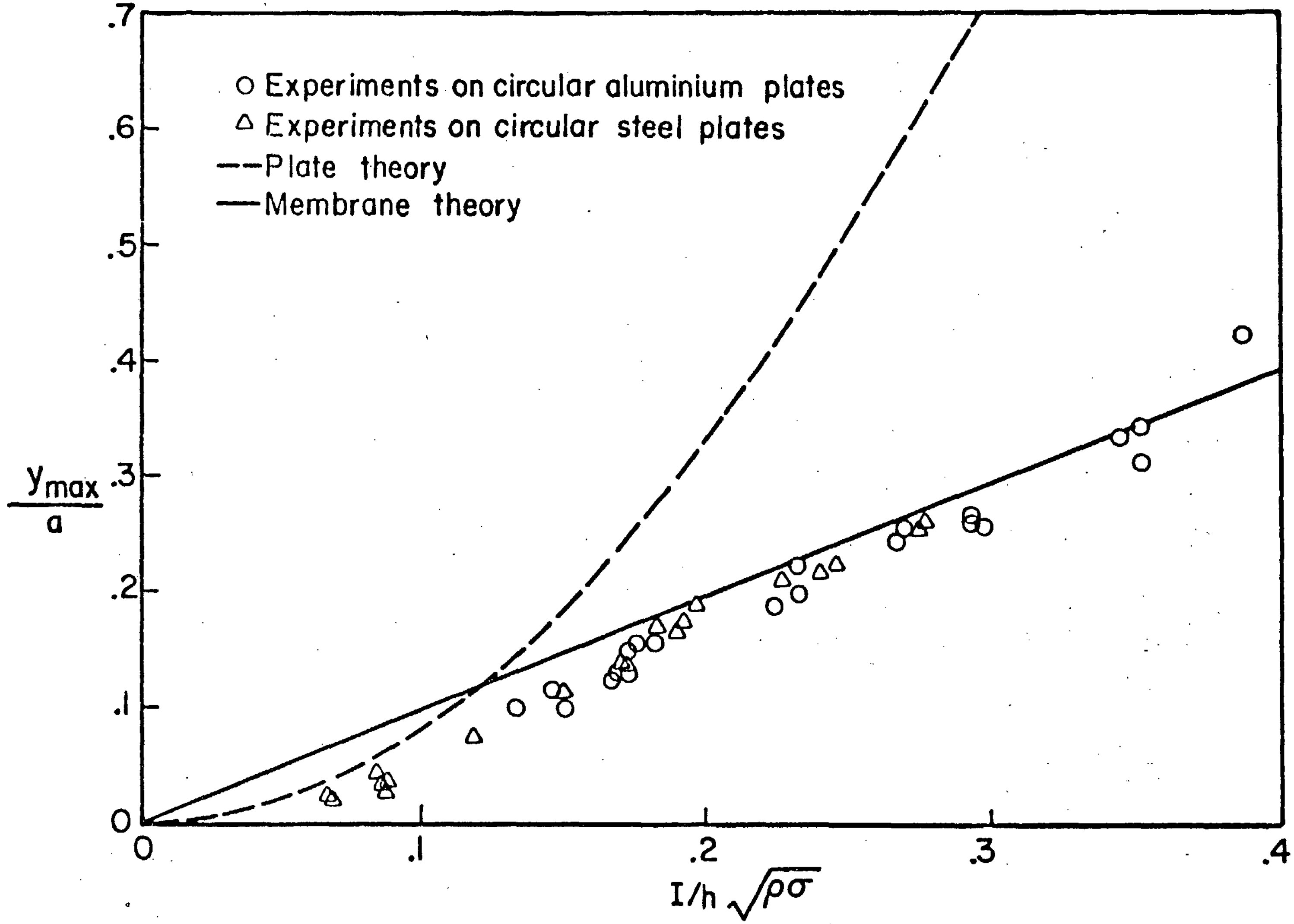


Fig. 8. The maximum deflection of a circular plate, subjected to a uniform impulse I, as determined by membrane theory (—), bending theory (--), and Florence's (1966) measurements.

$\eta$ and $\tau$ are defined by (3.3) and (3.4), and

$$A_i = \frac{a}{c\lambda_i} \frac{\int_0^1 v_0 J_0(\lambda_i \eta)\eta d\eta}{\int_0^1 J_0^2(\lambda_i \eta)\eta d\eta} = \frac{2v_0 a}{c\lambda_i^2 J_1(\lambda_i)} \quad . \tag{A.5}$$

The deflection at the center of the plate, r = 0, is given by

$$y_0/a = (v_0/c) f(\tau) \ , \tag{A.6}$$

where $$f(\tau) = 2 \sum_{i=1}^{\infty} [\lambda_i^2 J_1(\lambda_i)]^{-1} \sin(\lambda_i \tau) \quad . \tag{A.7}$$

Its first maximum (determined numerically) of 1.0 is the only one that has physical significance, since the formulation is valid only for increasing deflection.

The theoretical prediction $y_{max}/a = 1.0(v_0/c)$ is compared with Florence's results in Fig. 8. It is evident that the membrane model provides a significantly better correlation with the measured results than does the bending theory if $v_0/c > 0.15$ and that the correlation increases with $v_0/c$ within the range of the measurements.

APPENDIX II

LISTING OF COMPUTER PROGRAM

```
@FOR,IS    INT
FOR SE2C-03/24/75-13:22:42 (,0)

   MAIN PROGRAM

   STORAGE USED: CODE(1) 000362; DATA(0) 000100; BLANK COMMON(2) 000000

   COMMON BLOCKS:

   0003   EPSDEF 000002

   EXTERNAL REFERENCES (BLOCK, NAME)

   0004   CRF
   0005   XFORM
   0006   EXIT
   0007   NINTR$
   0010   NPRT$
   0011   NIO2$
   0012   CEXP
   0013   CDV$
   0014   NSTOP$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

   0001   000004 105G     0001   000105 131G     0000   000031 51CF     0000   000053 52CF     0000   000026 80CF
   0000 R 000012 DX       0000 R 000011 E        0003 C 000000 EPS      0000 C 000002 FF       0000 R 000016 F0
   0000 R 000017 F1       0000 R 000020 F2       0000 R 000021 F3       0000 I 000010 IE       0000 I 000014 IT
   0000 I 000013 K        0000 C 000006 Q        0000 C 000004 S        0000 C 000000 SSTRT    0000 R 000015 T
   0005 R 000000 XFORM    0000 R 000022 Z0       0000 R 000023 Z1       0000 R 000024 Z2       0000 R 000025 Z3

00101      1*           COMPLEX EPS,SSTRT,FF,S,Q                                000000
00103      2*           COMMON/EPSDEF/EPS                                       000004
00104      3*           DO 20 IE=1,1                                            000004
00107      4*           E=.25+0.                                                000004
00110      5*           PRINT 510,E                                             000006
00113      6*           DX=.0001                                                000013
00114      7*           EPS=CMPLX(E,0.)                                         000015
00115      8*           IF(IE.EQ.1) SSTRT=(-1.8E-1,1.4E-1)                      000020
00117      9*           IF(IE.NE.1)SSTRT=S                                      000025
00121     10*           CALL CRF(SSTRT,DX,FF,S,K)                               000032
00122     11*           PRINT 800,S,FF,K                                        000041
00127     12*       800 FORMAT (1P4E16.8,I8)                                    000105
00130     13*           DO 10 IT=1,501                                          000105
00133     14*           T=.0020*FLOAT(IT-1)                                     000105
00134     15*           F0=XFORM(T,E,0.)                                        000114
00135     16*           F1=XFORM(T,E,1.)                                        000122
00136     17*           F2=XFORM(T,E,2.)                                        000130
00137     18*           F3=XFORM(T,E,3.)                                        000136
```

```
00140        Q=CEXP(S*T)/(S**3+2.*E*S-E**2*S)               000144
00141        Z0=F0+2.*REAL(Q)                               000246
00142        Z1=-F1+2.*REAL(S*Q)                            000251
00143        Z2=F2+2.*REAL(S**2*Q)                          000272
00144        Z3=-F3+2.*REAL(S**3*Q)                         000313
00145        PRINT 520,T,F0,F1,F2,F3,Z0,Z1,Z2,Z3            000334
00160     10 CONTINUE                                       000355
00162     20 CONTINUE                                       000355
00164        CALL EXIT                                      000355
00165    510 FORMAT(4H1E =,1PE11.5,//,6X,1HT,12X,4HF(0),3X,4HF(1),   000361
00165      1  9X,4HF(2),9X,4HF(3),9X,4HZ(0),9X,4HZ(1),9X,4HZ(2),9X,4HZ(3))   000361
00166    520 FORMAT(1X,1P9E13.4)                            000361
00167        END                                            000361

END OF COMPILATION:        NO  DIAGNOSTICS.
```

```
@FOR,IS CICIP,CICIP
FOR SE2C-03/24/75-13:22:45 (,C)

   SUBROUTINE CICIP      ENTRY POINT 000202

   STORAGE USED: CODE(1) 000226; DATA(0) 000033; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   CDV$
    0004   NERR3$

   STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000032 113G      0000 C 000005 DENCI     0000 C 000010 DENCIP    0000 I 000007 I       0000    000020 INJP$
    0000 I 000004 NI        0000 C 000000 Y         0000 C 000002 YI

 00101      1*                 SUBROUTINE CICIP(Z,CI,CIP)                                    000000
 00103      2*                 IMPLICIT COMPLEX(A-H,O-Z)                                     000000
 00104      3*                 Y=0.25*Z**2                                                   000000
 00105      4*                 YI=1                                                          000017
 00106      5*                 NI=10                                                         000021
 00107      6*                 CI=1.0                                                        000023
 00110      7*                 CIP=1.0                                                       000024
 00111      8*                 DENCI=1.0                                                     000025
 00112      9*                 DO 11 I=1,NI                                                  000032
 00115     10*                 YI=Y*YI                                                       000032
 00116     11*                 DENCI=DENCI*I**2                                              000051
 00117     12*                 DENCIP=DENCI*(I+1)                                            000075
 00120     13*                 CI=CI+YI/DENCI                                                000121
 00121     14*                 CIP=CIP+YI/DENCIP                                             000132
 00122     15*              11 CONTINUE                                                      000145
 00124     16*                 CIP=CIP*0.5*Z                                                 000145
 00125     17*                 RETURN                                                        000167
 00126     18*                 END                                                           000225

        END OF COMPILATION:        NO DIAGNOSTICS.
```

```
@FOR,IS CKZK1,CKZK1
FOR SE2C-03/24/75-13:22:48 (,0)

    SUBROUTINE CKZK1     ENTRY POINT 000357

    STORAGE USED: CODE(1) 000415; DATA(0) 000060; BLANK COMMON(2) 000000

    EXTERNAL REFERENCES (BLOCK, NAME)

     0003   CICIP
     0004   CLOG
     0005   CDV$
     0006   NERR3$

    STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

     0001   000132 1236      0000 C 000011 CI      0000 C 000013 CIP      0000 C 000017 CKZP     0000 C 000015 CKZZP
     0000 C 000021 CL        0000 C 000023 DEN     0000 C 000000 GAMMA    0000 I 000025 I        0003   000041 INJP$
     0000 I 000002 NI        0000 C 000007 WS1     0000 C 000003 Y        0000 C 000005 YI

 00101      1*          SUBROUTINE CKZK1(Z,CKZ,CK1)                     000000
 00103      2*          IMPLICIT COMPLEX (A-H,O-Z)                      000000
 00104      3*          GAMMA=0.577215665                               000000
 00105      4*          NI=10                                           000001
 00106      5*          Y=0.25*Z**2                                     000003
 00107      6*          YI=1                                            000023
 00110      7*          WS1=CLOG(0.5*Z)+GAMMA                           000025
 00111      8*          CALL CICIP(Z,CI,CIP)                            000043
 00112      9*          CKZZP=-WS1*CIP-CI/Z                             000050
 00113     10*          CKZ=-CI*WS1                                     000100
 00114     11*          CKZP=0.                                         000120
 00115     12*          CL=0.                                           000122
 00116     13*          DEN=1.0                                         000123
 00117     14*          DO 11 I=1,NI                                    000132
 00122     15*          YI=YI*Y                                         000132
 00123     16*          CL=CL+1.0/I                                     000151
 00124     17*          DEN=DEN*I**2                                    000163
 00125     18*          CKZ=CKZ+YI/DEN*CL                               000207
 00126     19*          CKZP=CKZP+YI/Y/DEN*I*CL                         000235
 00127     20*       11 CONTINUE                                        000313
 00131     21*          CK1=-CKZZP-CKZP*0.5*Z                           000313
 00132     22*          RETURN                                          000342
 00133     23*          END                                             000414

      END OF COMPILATION:        NO DIAGNOSTICS.
```

```
@FOR,IS F.F
FOR SE2C-03/24/75-13:22:54 (,0)

     FUNCTION F          ENTRY POINT 000105

     STORAGE USED: CODE(1) 000120; DATA(0) 000012; BLANK COMMON(2) 000000

      COMMON BLOCKS:

      0003   EPSDEF 000002

     EXTERNAL REFERENCES (BLOCK, NAME)

      0004   CKZK1
      0005   NERR3$

     STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

      0000 C 000002 CKZ      0000 C 000004 CK1      0003 C 000000 EPS      0000 C 000000 F      0000   000006 INJP$


  00101      1*          COMPLEX FUNCTION F(Z)                                   000000
  00103      2*          IMPLICIT COMPLEX(A-H,O-Z)                               000000
  00104      3*          COMMON/EPSDEF/EPS                                       000000
  00105      4*          CALL CKZK1(/,CKZ,CK1)                                   000000
  00106      5*          F=Z**2*CKZ+EPS*Z*CK1                                    000004
  00107      6*          RETURN                                                  000074
  00110      7*          END                                                     000117

       END OF COMPILATION:      NO DIAGNOSTICS.
```

```
@FOR,IS CRF.CRF
FOR SE2C-03/24/75-13:22:55 (,0)

      SUBROUTINE CRF        ENTRY POINT 000267

      STORAGE USED: CODE(1) 000314; DATA(0) 000046; BLANK COMMON(2) 000000

      EXTERNAL REFERENCES (BLOCK, NAME)

        0003   F
        0004   NPRT$
        0005   NIO2$
        0006   NSTOP$
        0007   CDV$
        0010   CABS
        0011   NERR3$

      STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

        0000     000020 10F       0001    000011 21L       0000    000022 800F      0003 C 000000 F        0000 C 000016 FIM
        0000 C 000014 FIP         0000 C 000010 FRM        0000 C 000006 FRP        0000   000031 INJP$    0000 C 000002 X
        0000 R 000001 XM          0000 R 000000 XR         0000 C 000004 XS         0000 C 000012 XT


00101       1*            SUBROUTINE CRF(XSTRT,DX,FF,XX,K)                          000000
00103       2*            IMPLICIT COMPLEX(A-H,O-Z)                                 000000
00104       3*            REAL XR,XM,DX                                             000000
00105       4*            X=XSTRT                                                   000000
00106       5*            PRINT 10,X,XSTRT                                          000001
00112       6*         10 FORMAT(1P4E16.8)                                          000007
00113       7*            K=0                                                       000007
00114       8*         21 K=K+1                                                     000011
00115       9*            IF(K.GT.100)STOP KMAX                                     000013
00117      10*            XR=REAL(X)                                                000021
00120      11*            XM=AIMAG(X)                                               000023
00121      12*            XS=X                                                      000025
00122      13*            FF=F(X)                                                   000027
00123      14*            X=CMPLX(XR+DX,XM)                                         000033
00124      15*            FRP=F(X)                                                  000037
00125      16*            X=CMPLX(XR-DX,XM)                                         000043
00126      17*            FRM=F(X)                                                  000047
00127      18*            X=XS-FF*2.*DX/(FRP-FRM)                                   000053
00130      19*            XR=REAL(X)                                                000120
00131      20*            XM=AIMAG(X)                                               000121
00132      21*            FF=F(X)                                                   000123
00133      22*            XT=X                                                      000127
00134      23*            X=CMPLX(XR,XM+DX)                                         000131
00135      24*            FIP=F(X)                                                  000136
```

```
00136          X=CMPLX(XR,XM-DX)                              000142
00137          FIM=F(X)                                       000147
00140          X=XT-FF*2.*CMPLX(0.,DX)/(FIP-FIM)              000153
00141          PRINT 800,X,XT,XS,FF,FRP,FRM,FIP,FIM           000220
00153     800  FORMAT(1P8E14.7)                               000234
00154          IF(CABS(X-XS).GT.1.E-8)GO TO 21                000234
00155          FF=F(X)                                        000250
00157          XX=X                                           000254
00160          RETURN                                         000256
00161          END                                            000313

     END OF COMPILATION:        NO  DIAGNOSTICS.
```

```
@FOR,IS    BSSLIK
FOR U11A-10/29/73-12:38:19 (,0)

   SUBROUTINE BSSLIK    ENTRY POINT 000245

   STORAGE USED: CODE(1) 000336; DATA(0) 000124; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR6$
    0004   SQRT
    0005   EXP
    0006   ALOG
    0007   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000033 143G      0001   000071 161G      0001   000005 20L       0001   000143 202G      0001   000176 217G
    0001   000015 30L       0001   000056 50L       0001   000132 70L       0001   000163 90L       0000 R 000007 BI0
    0000 R 000027 BI1       0000 R 000047 BK0       0000 R 000065 BK1       0000 R 000103 EMX       0000 R 000104 EPX
    0000 R 000105 EPXLNT    0000   000115 INJP$     0000 I 000075 N         0000 R 000000 SI0       0000 R 000020 SI1
    0000 R 000040 SK0       0000 R 000056 SK1       0000 R 000076 SQRTX     0000 R 000074 T         0000 R 000100 TT
    0000 R 000101 XI0       0000 R 000102 XI1       0000 R 000077 XX


 00101            SUBROUTINE BSSLIK(X,XEMXI0,XEMXI1,XEPXK0,XEPXK1)
 00103            DIMENSION SI0(7),BI0(9),SI1(7),BI1(9),SK0(7),BK0(7),SK1(7),BK1(7)
 00104            DATA SI0/1., 3.5156229, 3.0899424, 1.2067492, .2659732, .0360768,
 00104          *          .0045813/
 00106            DATA BI0/.39894228, .01328592, .00225319, -.00157565, .00916281,
 00106          *          -.02057706, .02635537, -.01647633, .00392377/
 00110            DATA SI1/.5, .87890594, .51498869, .15084934, .02658733,
 00110          *          .00301532, .00032411/
 00112            DATA BI1/.39894228, -.03988024, -.00362018, .00163801, -.01031555,
 00112          *          .02282967, -.02895312, .01787654, -.00420059/
 00114            DATA SK0/-.57721566, .42278420, .23069756, .03488590, .00262698,
 00114          *          .00010750, .00000740/
 00116            DATA BK0/1.25331414, -.07832358, .02189568, -.01062446, .00587872,
 00116          *          -.00251540, .00053208/
 00120            DATA SK1/1., .15443144, -.67278579, -.18156897, -.01919402,
 00120          *          -.00110404, -.00004686/
 00122            DATA BK1/1.25331414, .23498619, -.03655620, .01504268, -.00780353,
 00122          *          .00325614, -.00068245/
 00124            IF(X)10,20,30
 00127         10 RETURN 0
 00130         20 XEMXI0=0.
 00131            XEMXI1=0.
 00132            XEPXK0=0.
```

```
BSSLIK                                                                    DATE 102973          PAGE    4

00133     24*         XEPXKI=1.
00134     25*         RETURN
00135     26*      30 IF(X.LT.3.75)GO TO 50
00137     27*         T=3.75/X
00140     28*         XEMXI0=BI0(9)
00141     29*         XEMXI1=BI1(9)
00142     30*         DO 40 N=8,1,-1
00145     31*         XEMXI0=XEMXI0*T+BI0(N)
00146     32*      40 XEMXI1=XEMXI1*T+BI1(N)
00150     33*         SQRTX=SQRT(X)
00151     34*         XEMXI0=SQRTX*XEMXI0
00152     35*         XEMXI1=SQRTX*XEMXI1
00153     36*         GO TO 70
00154     37*      50 XX=X*X
00155     38*         TT=XX/14.0625
00156     39*         XI0=SI0(7)
00157     40*         XI1=SI1(7)
00160     41*         DO 60 N=6,1,-1
00163     42*         XI0=XI0*TT+SI0(N)
00164     43*      60 XI1=XI1*TT+SI1(N)
00166     44*         XI0=X*XI0
00167     45*         XI1=XX*XI1
00170     46*         EMX=EXP(-X)
00171     47*         XEMXI0=EMX*XI0
00172     48*         XEMXI1=EMX*XI1
00173     49*         IF(X.LT.2.)GO TO 90
00175     50*         SQRTX=SQRT(X)
00176     51*      70 T=2./X
00177     52*         XEPXK0=BK0(7)
00200     53*         XEPXK1=BK1(7)
00201     54*         DO 80 N=6,1,-1
00204     55*         XEPXK0=XEPXK0*T+BK0(N)
00205     56*      80 XEPXK1=XEPXK1*T+BK1(N)
00207     57*         XEPXK0=SQRTX*XEPXK0
00210     58*         XEPXK1=SQRTX*XEPXK1
00211     59*         RETURN
00212     60*      90 T=X/2.
00213     61*         TT=T*T
00214     62*         XEPXK0=SK0(7)
00215     63*         XEPXK1=SK1(7)
00216     64*         DO 100 N=6,1,-1
00221     65*         XEPXK0=XEPXK0*TT+SK0(N)
00222     66*     100 XEPXK1=XEPXK1*TT+SK1(N)
00224     67*         EPX=EXP(X)
00225     68*         EPXLNT=EPX*ALOG(T)
00226     69*         XEPXK0=X*EPX*XEPXK0-EPXLNT*XI0
00227     70*         XEPXK1=EPX*XEPXK1+EPXLNT*XI1
00230     71*         RETURN
00231     72*         END

END OF COMPILATION:        NO DIAGNOSTICS.
```

```
@FOR,IS    DIENES
FOR U11A-10/29/73-12:38:27 (,0)

 FUNCTION DIENES      ENTRY POINT 000073

 STORAGE USED: CODE(1) 000104; DATA(0) 000022; BLANK COMMON(2) 000000

 EXTERNAL REFERENCES (BLOCK, NAME)

  0003   BSSLIK
  0004   EXP
  0005   NERR3$

 STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

  0000 R 000000 DIENES    0000 R 000002 E      0000   000012 INJP$   0000 R 000001 T     0000 R 000003 XI0
  0000 R 000004 XI1       0000 R 000005 XK0    0000 R 000006 XK1

00101      1*         FUNCTION DIENES(X,A)
00103      2*         DIMENSION A(1)
00104      3*         T=A(1)
00105      4*         E=A(2)
00106      5*         CALL BSSLIK(X,XI0,XI1,XK0,XK1)
00107      6*         DIENES=1./(EXP(X*(T-2.))*(X*XK0+E*XK1)**2
00107      7*        1  +9.8696044*EXP(X*(T+2.))*(X*XI0-E*XI1)**2)
00110      8*         RETURN
00111      9*         END

    END OF COMPILATION:       NO  DIAGNOSTICS.

@HDG      DNSMOD
```

```
@FOR,IS    D!!SMOD
FOR U11A-10/29/73-12:38:29 (,0)

    FUNCTION DNSMOD       ENTRY POINT 000021

    STORAGE USED: CODE(1) 000032; DATA(0) 000005; BLANK COMMON(2) 000000

    EXTERNAL REFERENCES (BLOCK, NAME)

     0003   DIENES
     0004   NERR3$

    STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

     0003 R 000000 DIENES    0000 R 000000 DNSMOD    0000    000001 INJPS

00101       1*          FUNCTION DNSMOD(X,A)
00103       2*          DIMENSION A(1)
00104       3*          DNSMOD=X**3*DIENES(X,A)
00105       4*          RETURN
00106       5*          END

      END OF COMPILATION:        NO  DIAGNOSTICS.

@HDG        CADRE
```

```
@FOR,IS   CADRE
FOR U11A-10/29/73-12:38:31 (,0)

FUNCTION CADRE        ENTRY POINT 002426

STORAGE USED: CODE(1) 002477; DATA(0) 005021; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

 0003   XPII
 0004   NWDUS
 0005   NI02$
 0006   NI01$
 0007   ALOG10
 0010   XPRI
 0011   NERR3$

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

 0001   000207 10L     0001   002226 1012G    0001   002241 1020G    0001   002352 1061G    0001   002361 1066G
 0001   000305 12L     0001   000465 15L      0001   000570 18L      0001   000617 20L      0001   000660 21L
 0001   000662 22L     0001   000253 221G     0001   000341 235G     0001   000432 255G     0001   000504 272G
 0001   000746 30L     0001   001015 31L      0001   001106 33L      0001   001166 333L     0001   001227 34L
 0001   001247 35L     0001   001261 36L      0001   001265 37L      0001   001370 40L      0001   001072 426G
 0001   001134 442G    0001   001147 450G     0001   001162 456G     0001   000122 5L       0001   001462 50L
 0001   001275 517G    0001   001360 544G     0001   000123 6L       0001   001535 60L      0000   004534 609F
 0000   004541 619F    0000   004545 620F     0001   001562 624G     0000   004552 629F     0000   004556 630F
 0000   004562 631F    0000   004565 632F     0000   004567 633F     0000   004606 638F     0000   004613 641F
 0000   004623 649F    0000   004627 650F     0000   004644 660F     0000   004651 667F     0000   004665 670F
 0000   004671 671F    0000   004700 682F     0000   004716 6900F    0000   004713 692F     0000   004731 6959F
 0001   001624 70L     0001   001770 700G     0001   001777 705G     0001   001657 71L      0001   001676 72L
 0001   002131 755G    0001   002140 762G     0001   001736 80L      0001   002010 82L      0001   002021 83L
 0001   002057 85L     0001   000166 9L       0001   002104 90L      0001   002306 900L     0001   002106 91L
 0001   002151 93L     0001   002165 94L      0001   002205 95L      0001   002325 950L     0001   002372 959L
 0001   002401 999L    0000 R 004512 ABS1     0000 R 000157 AIT      0000 L 004403 AITKEN   0000 R 004445 AITLOW
 0000 R 004447 AITTOL  0000 R 004453 ALG402   0000 R 004524 ALPHA    0000 R 004473 ASTEP    0000 R 004464 BEG
 0000 R 004246 BEGIN   0000 R 000000 CADRE    0000 R 004461 CUREST   0000 R 000171 DIF      0000 R 004521 DIFF
 0000 R 004467 END     0000 R 004516 ERGOAL   0000 R 004455 ERRA     0000 R 004520 ERRER    0000 R 004454 ERRR
 0000 R 004342 EST     0000 R 004465 FBEG     0000 R 004531 FBEG2    0000 R 004470 FEND     0000 R 004523 FEXTMI
 0000 R 004517 FEXTRP  0000 R 004304 FINIS    0000 R 004501 FN       0000 R 004462 FNSIZE   0000 R 004504 HOVN
 0000 L 004402 H2CONV  0000 R 004525 H2NEXT   0000 R 004527 H2TFEX   0000 R 004446 H2TOL    0000 I 004506 I
 0000 I 004466 IBEG    0000 I 004210 IBEGS    0000 I 004471 IEND     0000 I 004503 II       0000 I 004505 III
 0000   004767 INJP$   0000 I 004460 ISTAGE   0000 I 004502 ISTEP    0000 I 004507 ISTEP2   0000 I 004513 IT
 0000 I 004532 J       0000 R 004431 JUMPTL   0000 I 004475 L        0000 R 004400 LENGTH   0000 I 004477 LMI
 0000 I 004451 MAXTBL  0000 I 004450 MAXTS    0000 I 004452 MXSTGE   0000 I 004476 N        0000 I 004533 NNLEFT
 0000 I 004500 N2      0000 R 004463 PREVER   0000 R 000145 R        0000 L 004405 REGLAR   0000 L 004406 REGLSV
 0000 L 004404 RIGHT   0000 R 000203 RH       0000 R 004522 SING     0000 R 004526 SINGNX   0000 R 004530 SLOPE
 0000 R 004457 STAGE   0000 R 004472 STEP     0000 R 004456 STEPMN   0000 R 004510 SUM      0000 R 004511 SUMABS
 0000 R 000001 T       0000 R 004474 TABS     0000 R 004515 TABTLM   0000 R 004444 TOLMCH   0000 R 000207 TS
 0000 R 004514 VINT
```

```
00101          FUNCTION CADRE(F,ARG,A,B,AERR,RERR,LEVEL,ERROR,IFLAG)
00103          PARAMETER   Q10= 10,      Q30= 30,      Q2049= 2**(Q10+1) + 1
00104          DIMENSION T(Q10,Q10),R(Q10),AIT(Q10),DIF(Q10),RN(4),
00104         >   TS(Q2049),IBEGS(Q30),BEGIN(Q30),FINIS(Q30),EST(Q30),ARG(1)
00105          REAL LENGTH, JUMPTL
00106          LOGICAL H2CONV,AITKEN,RIGHT,REGLAR,REGLSV(Q30)
00107          DATA TOLMCH,AITLOW,H2TOL,AITTOL,JUMPTL,MAXTS,MAXTBL,MXSTGE
00107         *   / 1.E-07, 1.1 , .15 ,  .1  , .01  ,Q2049, Q10  , Q30/
00120          DATA RN/.71420053,.34662815,.843751,.12633046/
00122          DATA ALG402 /.301029996/
00124          CADRE = 0.
00125          ERROR = 0.
00126          IFLAG = 1
00127          LENGTH = ABS(B-A)
00130  *DIAGNOSTIC*  THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00130          IF (LENGTH .EQ. 0.)                 RETURN
00132          ERRR = AMIN1(.1,AMAX1(ABS(RERR), 10.*TOLMCH))
00133          ERRA = ABS(AERR)
00134          STEPMN = AMAX1(LENGTH/FLOAT(2**MXSTGE),
00134         *          AMAX1(LENGTH,ABS(A),ABS(B))*TOLMCH)
00135          STAGE = .5
00136          ISTAGE = 1
00137          CUREST = 0.
00140          FNSIZE = 0.
00141          PREVER = 0.
00142          REGLAR = .FALSE.
00143          BEG = A
00144          FBEG = F(BEG,ARG)/2.
00145          TS(1) = FBEG
00146          IBEG = 1
00147          END = B
00150          FEND = F(END,ARG)/2.
00151          TS(2) = FEND
00152          IEND = 2
00153        5 RIGHT = .FALSE.
00154        6 STEP = END - BEG
00155          ASTEP = ABS(STEP)
00156          IF (ASTEP .LT. STEPMN)              GO TO 950
00160          IF (LEVEL .GE. 3)  WRITE(6,609) BEG,STEP,ISTAGE
00166      609 FORMAT(10H BEG,STEP ,2E16.8,I5)
00167          T(1,1) = FBEG + FEND
00170          TABS = ABS(FBEG) + ABS(FEND)
00171          L = 1
00172          N = 1
00173          H2CONV = .FALSE.
00174          AITKEN = .FALSE.
00175                                              GO TO 10
00176        9 IF (LEVEL .GE. 4) WRITE(6,692) L,T(1,LM1)
00203       10 LM1 = L
00204          L = L + 1
00205          N2 = N*2
00206          FN = N2
00207          ISTEP = (IEND - IBEG)/N
```

```
00210          IF (ISTEP .GT. 1)                   GO TO 12
00212          II = IEND
00213          IEND = IEND + N
00214          IF (IEND .GT. MAXTS)                GO TO 900
00216          HOVN = STEP/FN
00217          III = IEND
00220          DO 11 I=1,N2,2
00223          TS(III) = TS(II)
00224          TS(III-1) = F(END - FLOAT(I)*HOVN , ARG)
00225          III = III-2
00226       11 II = II-1
00230          ISTEP = 2
00231       12 ISTEP2 = IBEG + ISTEP/2
00232          SUM = 0.
00233          SUMABS = 0.
00234          DO 13 I=ISTEP2,IEND,ISTEP
00237          SUM = SUM + TS(I)
00240       13 SUMABS = SUMABS + ABS(TS(I))
00242          T(L,1) = T(L-1,1)/2. + SUM/FN
00243          TABS = TABS/2. + SUMABS/FN
00244          ABSI = ASTEP*TABS
00245          N = N2
00246          IT = 1
00247          VINT = STEP*T(L,1)
00250          TABTLM = TABS*TOLMCH
00251          FNSIZE = AMAX1(FNSIZE,ABS(T(L,1)))
00252          ERGOAL = AMAX1(ASTEP*TOLMCH*FNSIZE,
00252         *       STAGE*AMAX1(ERRA,ERRR*ABS(CUREST+VINT)))
00252     COMPLETE ROW L AND COLUMN L OF *T* ARRAY.
00253          FEXTRP = 1.
00254          DO 14 I=1,LM1
00257          FEXTRP = FEXTRP*4.
00260          T(I,L) = T(L,I) - T(L-1,I)
00261       14 T(L,I+1) = T(L,I) + T(I,L)/(FEXTRP-1.)
00263          ERRER = ASTEP*ABS(T(1,L))
00264          IF (L .GT. 2)                       GO TO 15
00266          IF (ABS(T(1,2)) .LE. TABTLM)        GO TO 60
00270                                              GO TO 10
00270     CALCULATE NEXT RATIOS FOR COLUMNS 1,.....,L-2 OF T-TABLE
00271       15 DO 16 I=2,LM1
00274          DIFF = 0.
00275          IF (ABS(T(I-1,L)) .GT. TABTLM) DIFF = T(I-1,LM1)/T(I-1,L)
00277       16 T(I-1,LM1) = DIFF
00301          IF (ABS(4.-T(1,LM1)) .LE. H2TOL) GO TO 20
00303 *DIAGNOSTIC*  THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00303          IF (T(1,LM1) .EQ. 0.)               GO TO 18
00305          IF (ABS(2.-ABS(T(1,LM1))) .LT. JUMPTL) GO TO 50
00307          IF (L .EQ. 3)                       GO TO 9
00311          H2CONV = .FALSE.
00312          IF (ABS(T(1,LM1)-T(1,L-2)) .LE. AITTOL*ABS(T(1,LM1)))
00312         *                                    GO TO 30
00314       17 IF (REGLAR)                         GO TO 18
00316          IF (L .EQ. 4)                       GO TO 9
00320       18 IF (ERRER .LE. ERGOAL)              GO TO 70
```

```
00322             IF (LEVEL .GE. 4)  WRITE (6,692) L,T(1,LM1)
00327                                             GO TO 91
00327       CAUTIOUS ROMBERG EXTRAPOLATION  -----------------------------------------
00330          20 IF (LEVEL .GE. 4) WRITE (6,619) L,T(1,LM1)
00335         619 FORMAT(I5,E16.8,5X6HH2CONV)
00336             IF (H2CONV)                     GO TO 21
00340             AITKEN = .FALSE.
00341             H2CONV = .TRUE.
00342             IF (LEVEL .GE. 3)  WRITE (6,620) L
00346         620 FORMAT(22H H2 CONVERGENCE AT ROW,I3)
00347          21 FEXTRP = 4.
00350          22 IT = IT + 1
00351             VINT = STEP*T(L,IT)
00352             ERRER = ABS(STEP/(FEXTRP-1.)*T(IT-1,L))
00353             IF (ERRER .LE. ERGOAL)          GO TO 80
00355             IF (IT .EQ. LM1)                GO TO 40
00357 *DIAGNOSTIC*  THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00357             IF (T(IT,LM1) .EQ. 0.)          GO TO 22
00361             IF (T(IT,LM1) .LE. FEXTRP)      GO TO 40
00363             IF (ABS(T(IT,LM1)/4.-FEXTRP) .LT. FEXTRP*AITTOL)
00363            *            FEXTRP = FEXTRP*4.
00365                                             GO TO 22
00366          30 IF (LEVEL .GE. 4) WRITE (6,629) L,T(1,LM1)
00373         629 FORMAT(I5,E16.8,5X6HAITKEN)
00374             IF (T(1,LM1) .LT. AITLOW)       GO TO 91
00376             IF (AITKEN)                     GO TO 31
00400             H2CONV = .FALSE.
00401             AITKEN = .TRUE.
00402             IF (LEVEL .GE. 3) WRITE (6,630) L
00406         630 FORMAT(14H AITKEN AT ROW,I3)
00407          31 FEXTRP = T(L-2,LM1)
00410             IF (FEXTRP .GT. 4.5)            GO TO 21
00412             IF (FEXTRP .LT. AITLOW)         GO TO 91
00414             IF (ABS(FEXTRP-T(L-3,LM1)) .GT. T(1,LM1)*H2TOL)
00414            *                                GO TO 91
00416             IF (LEVEL .GE. 3) WRITE (6,631) FEXTRP
00422         631 FORMAT(6H RATIO,F12.8)
00423             SING = FEXTRP
00424             FEXTM1 = FEXTRP - 1.
00425             DO 32 I=2,L
00430             AIT(I) = T(I,1) + (T(I,1)-T(I-1,1))/FEXTM1
00431             R(I) = T(1,I-1)
00432          32 DIF(I) = AIT(I) - AIT(I-1)
00434             IT = 2
00435          33 VINT = STEP*AIT(L)
00436             IF (LEVEL .LT. 5) GO TO 333
00440             WRITE (6,632) (R(I+1),I=IT,LM1)
00446             WRITE( 6,632) (AIT(I),I=IT,L)
00454             WRITE (6,632) (DIF(I+1),I=IT,LM1)
00462         632 FORMAT(1X,8E15.8)
00463         333 ERRER = ERRER/FEXTM1
00464             IF (ERRER .GT. ERGOAL)          GO TO 34
00466             ALPHA = ALOG10(SING)/ALG402 - 1.
00467             IF (LEVEL .GE. 2) WRITE (6,633) ALPHA,BEG,END
```

```
00475      633 FORMAT(11X42HINTEGRAND SHOWS SINGULAR BEHAVIOR OF TYPE
00475         *         4HX**(F4.2,9H) BETWEENE15.8,4H ANDE15.8)
00476          IFLAG = MAXO(IFLAG,2)
00477                                               GO TO 80
00500       34 IT = IT + 1
00501          IF (IT .EQ. LM1)                     GO TO 40
00503          IF (IT .GT.3)                        GO TO 35
00505          H2NEXT = 4.
00506          SINGNX = SING+SING
00507       35 IF (H2NEXT .LT. SINGNX)              GO TO 36
00511          FEXTRP = SINGNX
00512          SINGNX = SINGNX+SINGNX
00513                                               GO TO 37
00514       36 FEXTRP = H2NEXT
00515          H2NEXT = 4.*H2NEXT
00516       37 DO 38 I=IT,LM1
00521          R(I+1) = 0.
00522       38 IF (ABS(DIF(I+1)) .GT. TABTLM) R(I+1) = DIF(I)/DIF(I+1)
00525          IF (LEVEL .GE. 4) WRITE (6,638) FEXTRP,R(L-1),R(L)
00533      638 FORMAT(16H FEXTRP + RATIOS,3E15.8)
00534          H2TFEX = -H2TOL*FEXTRP
00535          IF (R(L)   .LT. H2TFEX+FEXTRP)   GO TO 40
00537          IF (R(L-1) .LT. H2TFEX+FEXTRP)   GO TO 40
00541          ERRER = ASTEP*ABS(DIF(L))
00542          FEXTM1 = FEXTRP - 1.
00543          DO 39 I=IT,L
00546          AIT(I) = AIT(I) + DIF(I)/FEXTM1
00547       39 DIF(I) = AIT(I) - AIT(I-1)
00551                                               GO TO 33
00552       40 FEXTRP = AMAX1(PREVER/ERRER,AITLOW)
00553          PREVER = ERRER
00554          IF (L .LT. 5)                        GO TO 10
00556          IF (LEVEL .GE. 3) WRITE (6,641) ERRER,ERGOAL,FEXTRP,IT
00565      641 FORMAT(23H ERRER,ERGOAL,FEXTRP,IT,2E15.8,F14.5,I3)
00566          IF (L-IT .GT. 2 .AND. ISTAGE .LT. MXSTGE) GO TO 90
00570          IF (ERRER .LT. ERGOAL*FEXTRP**(MAXTBL-L)) GO TO 10
00572                                               GO TO 90
00573       50 IF (LEVEL .GE. 4) WRITE (6,649) L,T(1,LM1)
00600      649 FORMAT(I5,E16.8,5X4HJUMP)
00601          IF (ERRER .GT. ERGOAL)               GO TO 90
00603          DIFF = ABS(T(1,L))*(FN + FN)
00604          IF (LEVEL .GE. 2) WRITE (6,650) DIFF,BEG,END
00612      650 FORMAT(13X36HINTEGRAND SEEMS TO HAVE JUMP OF SIZEE13.6,
00612         *         8H BETWEENE15.8,4H ANDE15.8)
00613                                               GO TO 80
00614       60 IF (LEVEL .GE. 4) WRITE (6,660) L
00620      660 FORMAT(I5,21X,13HSTRAIGHT LINE)
00621          SLOPE = (FEND-FBEG) + (FEND-FBEG)
00622          FBEG2 = FBEG+FBEG
00623          DO 61 I=1,4
00626          DIFF = ABS(F(BEG+RN(I)*STEP , ARG) - FBEG2-RN(I)*SLOPE)
00627          IF (DIFF .GT. TABTLM)                GO TO 72
00631       61 CONTINUE
00633          IF (LEVEL .GE. 3)  WRITE (6,667) BEG, END
```

```
00640       667 FORMAT(27X43HINTEGRAND SEEMS TO BE STRAIGHT LINE BETWEEN
00640          •        E15.8,4H ANDE15.8)
00641                                                  . GO TO 80
00642        70 IF (LEVEL .GE. 4) WRITE (6,670) L,T(1,LM1)
00647       670 FORMAT(I5,E16.8,5X5HNOISE)
00650           SLOPE = (FEND-FBEG) + (FEND-FBEG)
00651           FBEG2 = FBEG+FBEG
00652           I = 1
00653        71 DIFF = ABS(F(BEG+RN(I)*STEP , ARG) - FBEG2-RN(I)*SLOPE)
00654        72 ERRER = AMAX1(ERRER,ASTEP*DIFF)
00655           IF (EPRER .GT. ERGOAL)              GO TO 91
00657           I = I+1
00660           IF (I .LE. 4)                       GO TO 71
00662           IF (LEVEL .GE. 3) WRITE (6,671) BEG,END
00667       671 FORMAT(15H NOISE BETWEEN ,E15.8,4H AND,E15.8)
00670           IFLAG = 3
00671        80 CADRE = CADRE + VINT
00672           ERROR = ERROR + ERRER
00673           IF (LEVEL .LT. 3)                   GO TO 83
00675           IF (LEVEL .LT. 5)                   GO TO 82
00677           DO 81 I=1,L
00702        81 WRITE (6,692) I,(T(I,J),J=1,L)
00712        82 WRITE (6,682) VINT,ERRER,L,IT
00720       682 FORMAT(12H INTEGRAL IS,E16.8,7H, ERROR,E15.8,9H  FROM T(,
00720          •        I1,1H,,I1,1H))
00721        83 IF (RIGHT)                          GO TO 85
00723           ISTAGE = ISTAGE - 1
00724           IF (ISTAGE .EQ. 0)                  RETURN
00726           REGLAR = REGLSV(ISTAGE)
00727           BEG = BEGIN(ISTAGE)
00730           END = FINIS(ISTAGE)
00731           CUREST = CUREST - EST(ISTAGE+1) + VINT
00732           IEND = IBEG - 1
00733           FEND = TS(IEND)
00734           IBEG = IBEGS(ISTAGE)
00735                                               GO TO 94
00736        85 CUREST = CUREST + VINT
00737           STAGE = STAGE+STAGE
00740           IEND = IBEG
00741           IBEG = IBEGS(ISTAGE)
00742           END = BEG
00743           BEG = BEGIN(ISTAGE)
00744           FEND = FBEG
00745           FBEG = TS(IBEG)
00746                                               GO TO 5
00747        90 REGLAR = .TRUE.
00750        91 IF (ISTAGE .EQ. MXSTGE)             GO TO 950
00752           IF (LEVEL .LT. 5)                   GO TO 93
00754           DO 92 I=1,L
00757        92 WRITE (6,692) I,(T(I,J),J=1,L)
00767       692 FORMAT(I5,7E16.8/3E16.8)
00770        93 IF (RIGHT)                          GO TO 95
00772           REGLSV(ISTAGE+1) = REGLAR
00773           BEGIN(ISTAGE) = BEG
```

```
00774          IBEGS(ISTAGE) = IBEG
00775          STAGE = STAGE/2.
00776       94 RIGHT = .TRUE.
00777          BEG = (BEG+END)/2.
01000          IBEG = (IBEG+IEND)/2
01001          TS(IBEG) = TS(IBEG)/2.
01002          FBEG = TS(IBEG)
01003                                          GO TO 6
01004       95 NNLEFT = IBEG - IBEGS(ISTAGE)
01005          IF (IEND+NNLEFT .GE. MAXTS)     GO TO 900
01007          III = IBEGS(ISTAGE)
01010          II = IEND
01011          DO 96 I=III,IBEG
01014          II = II +1
01015       96 TS(II) = TS(I)
01017          DO 97 I=IBEG,II
01022          TS(III) = TS(I)
01023       97 III = III + 1
01025          IEND = IEND + 1
01026          IBEG = IEND - NNLEFT
01027          FEND = FBEG
01030          FBEG = TS(IBEG)
01031          FINIS(ISTAGE) = END
01032          END = BEG
01033          BEG = BEGIN(ISTAGE)
01034          BEGIN(ISTAGE) = END
01035          REGLSV(ISTAGE) = REGLAR
01036          ISTAGE = ISTAGE + 1
01037          REGLAR = REGLSV(ISTAGE)
01040          EST(ISTAGE) = VINT
01041          CUREST = CUREST + EST(ISTAGE)
01042                                          GO TO 5
01043      900 IF (LEVEL .GE. 2) WRITE (6,6900) BEG, END
01050     6900 FORMAT(37H TOO MANY FUNCTION EVALUATIONS AROUND/
01050        .          10X,E15.8,4H AND,E15.8)
01051          IFLAG = 4
01052                                          GO TO 999
01053      950 IFLAG = 5
01054          IF (LEVEL .LT. 2)               GO TO 999
01056          IF (LEVEL .LT. 5)               GO TO 959
01060          DO 958 I=1,L
01063      958 WRITE (6,692) I,(T(I,J),J=1,L)
01073      959 WRITE (6,6959) BEG, END
01077     6959 FORMAT(12X38HINTEGRAND SHOWS SINGULAR BEHAVIOUR OF
01077        .          20HUNKNOWN TYPE BETWEENE15.8,4H ANDE15.8)
01100      999 CADRE = CUREST + VINT
01101                                          RETURN
01102          END
```

END OF COMPILATION:          3 DIAGNOSTICS.

DISTRIBUTION LIST

| No. of Copies | Organization |
|---|---|
| 12 | Commander<br>Defense Documentation Center<br>ATTN: DDC-TCA<br>Cameron Station<br>Alexandria, VA 22314 |
| 1 | Director of Defense<br>Research & Engineering (OSD)<br>Washington, DC 20301 |
| 1 | Director<br>Defense Advanced Research<br>Projects Agency<br>1400 Wilson Boulevard<br>Arlington, VA 22209 |
| 1 | Director<br>Weapons Systems Evaluation<br>Group<br>Washington, DC 20305 |
| 1 | Chairman<br>Defense Science Board<br>Washington, DC 20301 |
| 1 | Commander<br>US Army Materiel Development<br>and Readiness Command<br>ATTN: DRCDMA-ST<br>5001 Eisenhower Avenue<br>Alexandria, VA 22333 |
| 1 | Commander<br>US Army Aviation Systems<br>Command<br>ATTN: DRSAV-E<br>12th and Spruce Streets<br>St. Louis, MO 61366 |
| 1 | Director<br>US Army Air Mobility Research<br>and Development Laboratory<br>Ames Research Center<br>Moffett Field, CA 94035 |

| No. of Copies | Organization |
|---|---|
| 1 | Commander<br>US Army Electronics Command<br>ATTN: DRSEL-RD<br>Fort Monmouth, NJ 07703 |
| 1 | Director<br>Research and Development<br>Technical Support Agency<br>ATTN: DRSEL-GG-E, Mr. Goldy<br>US Army Electronics Command<br>Fort Monmouth, NJ 07703 |
| 1 | Commander<br>US Army Tank Automotive<br>Development Command<br>ATTN: DRDTA-RWL<br>Warren, MI 48090 |
| 1 | Commander<br>US Army Missile Command<br>ATTN: DRSMI-R<br>Redstone Arsenal, AL 35809 |
| 2 | Commander<br>US Army Mobility Equipment<br>Research & Development Command<br>ATTN: Tech Docu Cen, Bldg. 315<br>DRSME-RZT<br>Fort Belvoir, VA 22060 |
| 1 | Commander<br>US Army Armament Command<br>Rock Island, IL 61202 |
| 1 | Commander<br>US Army Picatinny Arsenal<br>ATTN: SARPA-DW-6<br>Dover, NJ 07801 |
| 2 | Commander<br>US Army Frankford Arsenal<br>ATTN: SARFA-J7400<br>SARFA-J7600<br>Philadelphia, PA 19137 |

DISTRIBUTION LIST

| No. of Copies | Organization |
|---|---|
| 1 | Commander<br>US Army Harry Diamond Labs<br>ATTN: DRXDO-TI<br>2800 Powder Mill Road<br>Adelphi, MD 20783 |
| 4 | Commander<br>US Army Materials and<br>Mechanics Research Center<br>ATTN: Dr. R. Frost<br>Mr. A. Alesi<br>Mr. K. Abbott<br>Mr. E. DeLuca<br>Watertown, MA 02172 |
| 6 | Commander<br>US Army Natick Research and<br>Development Center<br>ATTN: Mr. T. Bailey<br>Mr. T. Keville<br>Mr. W. Wright<br>Mr. F. Figucia<br>Mr. M. Denomee<br>MAJ W. R. Belcher<br>Natick, MA 01760 |
| 1 | Director<br>US Army TRADOC Systems<br>Analysis Activity<br>ATTN: ATAA-SA<br>White Sands Missile Range<br>NM 88002 |
| 1 | Commander<br>US Army Training and<br>Doctrine Command<br>ATTN: MAJ R. Young<br>Fort Monroe, VA 23651 |
| 1 | Commander<br>US Army Armor School<br>ATTN: Armor Agency<br>Fort Knox, KY 40121 |

| No. of Copies | Organization |
|---|---|
| 1 | Commander<br>US Army Aviation School<br>ATTN: Aviation Agency<br>Fort Rucker, AL 36362 |
| 1 | Commander<br>US Army Field Artillery School<br>ATTN: Field Artillery Agency<br>Fort Sill, OK 73405 |
| 1 | The Surgeon General<br>Department of the Army<br>Washington, DC 20315 |
| 1 | Chief of Naval Operations<br>ATTN: OP-03EG<br>Department of the Navy<br>Washington, DC 20350 |
| 1 | Commander<br>US Naval Air Systems Command<br>ATTN: AIR-604<br>Washington, DC 20360 |
| 2 | Commander<br>US Naval Weapons Center<br>ATTN: Code 753<br>Code 12<br>China Lake, CA 93555 |
| 1 | Commander<br>US Naval Medical Field<br>Research Laboratories<br>Camp LeJuene, NC 28542 |
| 1 | Commander<br>US Naval Research Laboratories<br>ATTN: Code 8434, W. Ferguson<br>Washington, DC 20375 |
| 1 | Commandant<br>US Marine Corps<br>Washington, DC 20380 |

DISTRIBUTION LIST

| No. of Copies | Organization |
|---|---|
| 1 | Director, MCDED<br>ATTN: MAJ D. Crowe<br>Quantico, VA 22134 |
| 1 | AFATL (DLRW)<br>Eglin AFB, FL 32542 |
| 1 | US Department of Commerce<br>National Bureau of Standards<br>Law Enforcement Standards Lab<br>Washington, DC 20236 |

Aberdeen Proving Ground

Cdr, USAEA
  ATTN: SAREA-BL-B
Dir, USAMSAA
Dir, USAHEL
  ATTN: Mr. B. Corona
Marine Corps Ln Ofc